TOPIC: AREA BETWEEN CURVES

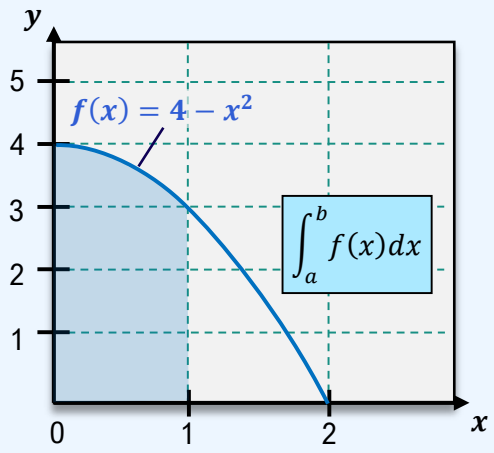
Finding Area Between Curves on a Given Interval

◆ To find the area **between** two curves, *ALWAYS* graph the functions & shade the area between them *first*.

► Then, integrate the _____ function **minus** the _____ function.

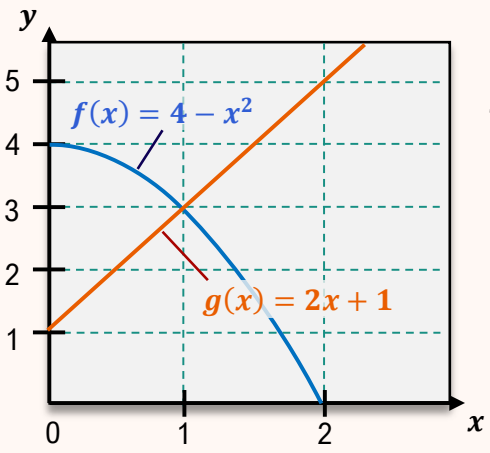
EXAMPLE Find the area of the region between $f(x)$ & $g(x)$ from $x = 0$ to $x = 1$.

Recall **Area *Under* Curve**



$$\text{Area} = \int_0^1 (4 - x^2)dx = \frac{11}{3}$$

New **Area *Between* Curves**



$$\int_a^b \_\_(x)dx - \int_a^b \_\_(x)dx$$

TOP

$$\int_a^b [f(x) \quad g(x)]dx$$

BOTTOM

Area =

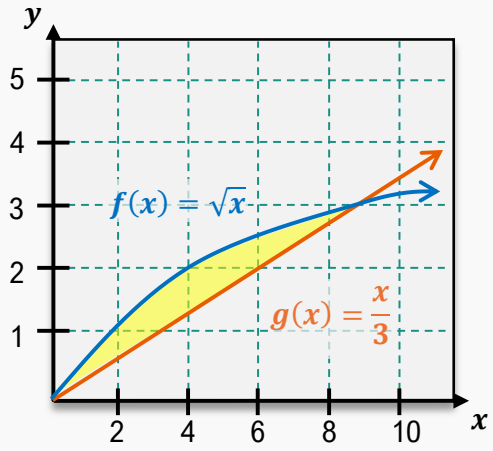
◆ Note: This method works even when function(s) are below the x-axis.

TOPIC: AREA BETWEEN CURVES

PRACTICE Calculate the area of the shaded region between the 2 functions from $x = 0$ to $x = 9$.

Recall

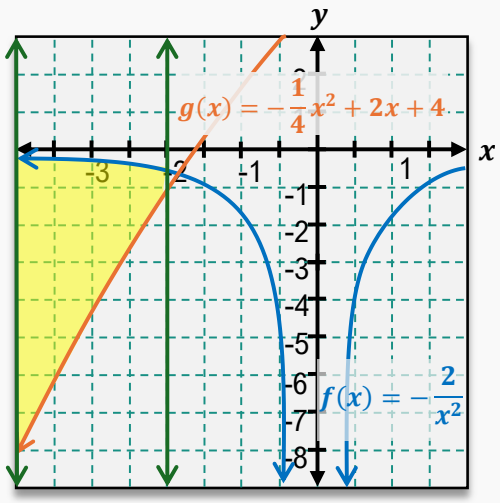
$$A = \int_a^b [f(x) - g(x)]dx$$



PRACTICE Calculate the area of the shaded region between $f(x)$ & $g(x)$ contained between $x = -4$ & $x = -2$.

Recall

$$A = \int_a^b [f(x) - g(x)]dx$$



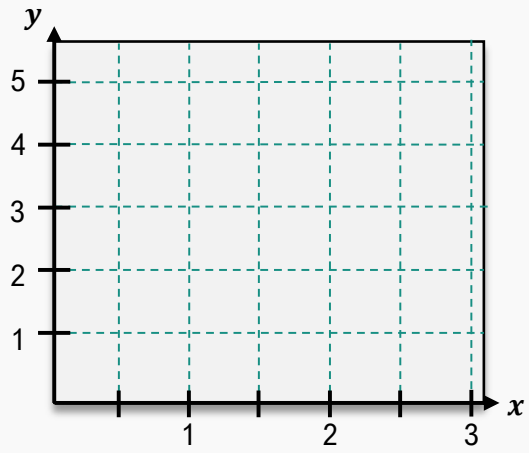
TOPIC: AREA BETWEEN CURVES

PRACTICE

Sketch the region bounded by $f(x) = -(x-2)^2 + 5$ & $g(x) = 4x$ on the interval $[0,1]$. Then set up an integral to represent the region's area and evaluate.

Recall

$$A = \int_a^b [f(x) - g(x)]dx$$

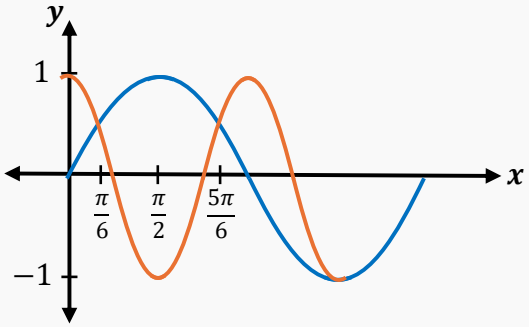


PRACTICE

Shade the region bounded by $f(x) = \sin x$ & $g(x) = \cos 2x$ on the interval $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$. Then set up an integral to represent the region's area.

Recall

$$A = \int_a^b [f(x) - g(x)]dx$$

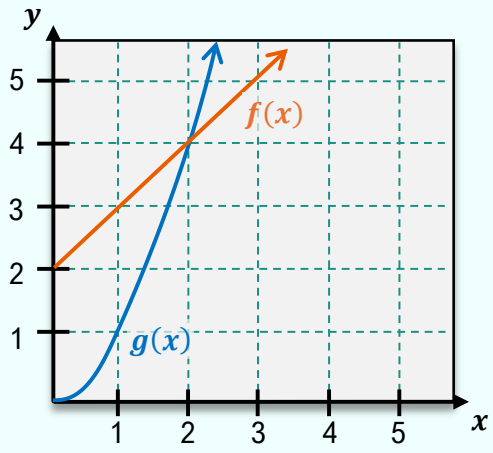


TOPIC: AREA BETWEEN CURVES

EXAMPLE Find the area between $f(x) = |x| + 2$ & $g(x) = x^2$ from $x = 0$ to $x = 2$.

Recall

$$A = \int_a^b [f(x) - g(x)]dx$$



TOPIC: AREA BETWEEN CURVES

Finding Area When Bounds Are Not Given

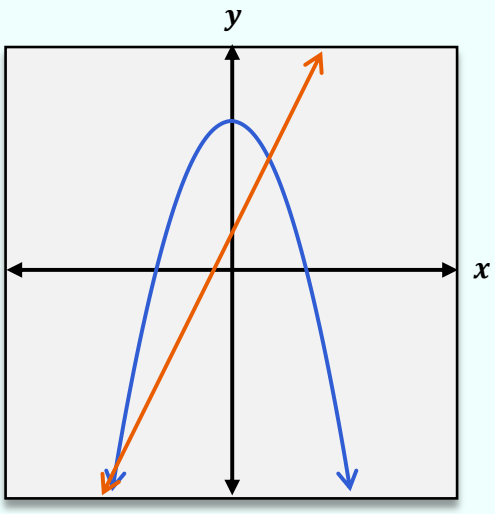
◆ If bounds are *not* given, use the _____ points. To find them, set the functions equal & solve for x.

EXAMPLE Find the area between $y = 4 - x^2$ & $y = 2x + 1$.

Recall

$$A = \int_a^b [f(x) - g(x)]dx$$

(Area Between Curves)

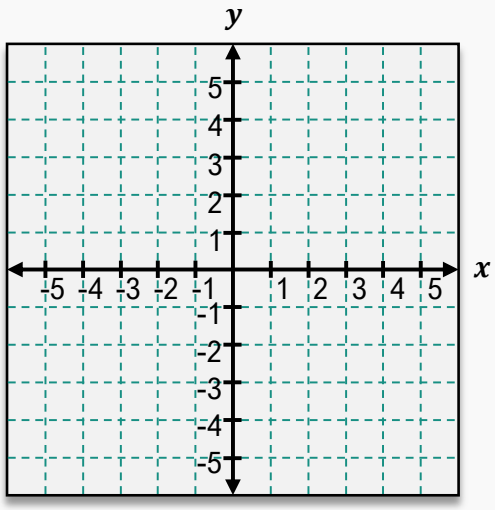


TOPIC: AREA BETWEEN CURVES

PRACTICE Find the area between $f(x) = x^2 - 4$ & $g(x) = -x^2 + 4$.

Recall

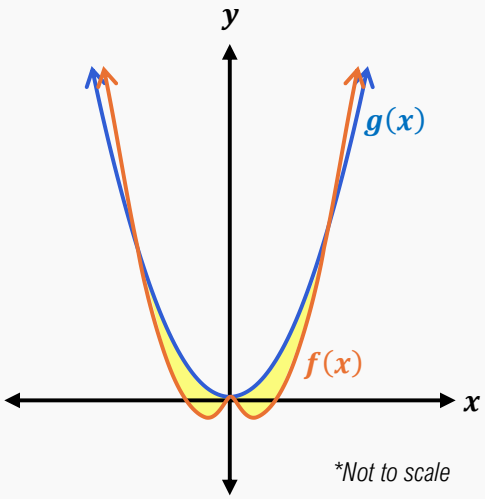
$$A = \int_a^b [f(x) - g(x)]dx$$



PRACTICE Find the area of the shaded region between $f(x) = x^4 - x^2$ & $g(x) = 3x^2$.

Recall

$$A = \int_a^b [f(x) - g(x)]dx$$

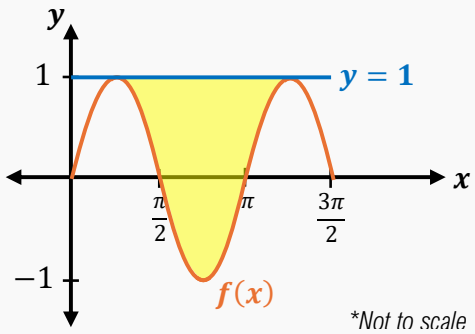


TOPIC: AREA BETWEEN CURVES

PRACTICE Find the area of the shaded region *ONLY* that lies between the line $y = 1$ & $f(x) = \sin 2x$.

Recall

$$A = \int_a^b [f(x) - g(x)]dx$$



*Not to scale

TOPIC: AREA BETWEEN CURVES

Finding Area Between Curves That Cross on the Interval

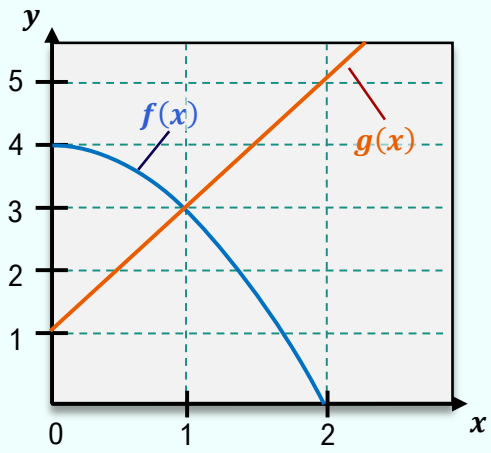
◆ To find the area between curves that **intersect** within the given interval, set up ____ integrals & _____ them.

► Both integrals will still be TOP – BOTTOM, but the top & bottom functions _____ after the intersection.

EXAMPLE Set up an integral to represent the area between $f(x) = 4 - x^2$ & $g(x) = 2x + 1$ on $[0,2]$.

New

$$\int_a^b [f(x) - g(x)]dx + \int_b^c [\_\_(x) - \_\_(x)]dx$$



TOPIC: AREA BETWEEN CURVES

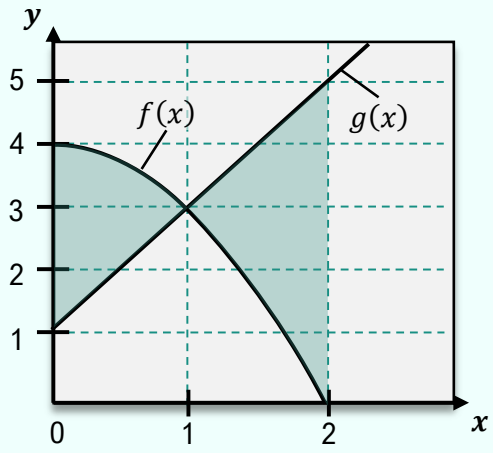
EXAMPLE Calculate the area between $f(x) = 4 - x^2$ & $g(x) = 2x + 1$ on $[0,2]$.

Recall

$$A = \int_a^b [f(x) - g(x)]dx + \int_b^c [g(x) - f(x)]dx$$

(Area Between Curves w/ Intersection)

$$A = \int_0^1 [(4 - x^2) - (2x + 1)]dx + \int_1^2 [(2x + 1) - (4 - x^2)]dx$$

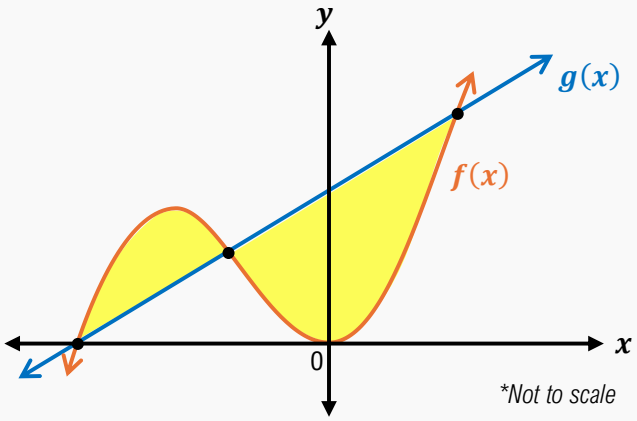


TOPIC: AREA BETWEEN CURVES

PRACTICE Find the shaded area between $f(x) = x^3 + 2x^2$ & $g(x) = x + 2$.

Recall

$$A = \int_a^b [f(x) - g(x)]dx + \int_b^c [\,g(x) - f(x)]dx$$

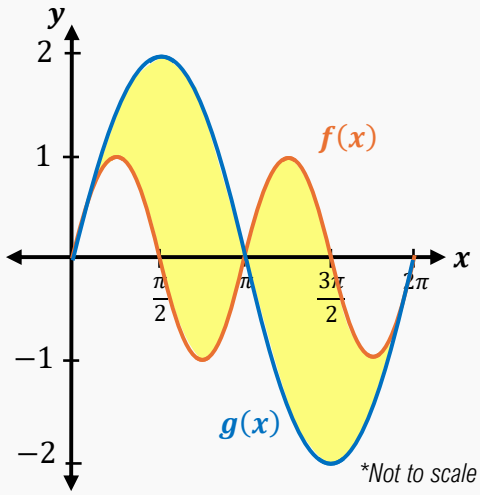


TOPIC: AREA BETWEEN CURVES

PRACTICE Find the area of the shaded region between $f(x) = \sin 2x$ & $g(x) = 2\sin x$ from $x = 0$ to $x = 2\pi$.

Recall

$$A = \int_a^b [f(x) - g(x)]dx + \int_b^c [g(x) - f(x)]dx$$



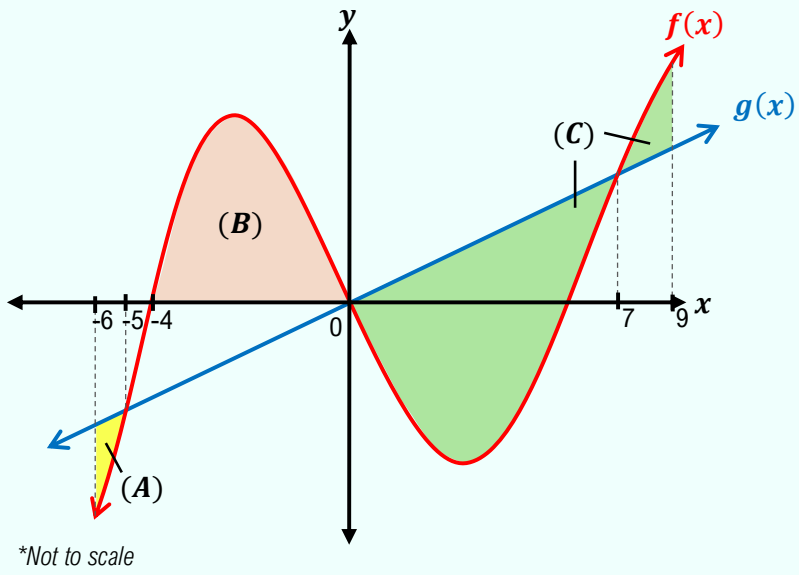
TOPIC: AREA BETWEEN CURVES

EXAMPLE Set up a definite integral to represent each of the 3 shaded areas.

(A)

(B)

(C)



*Not to scale