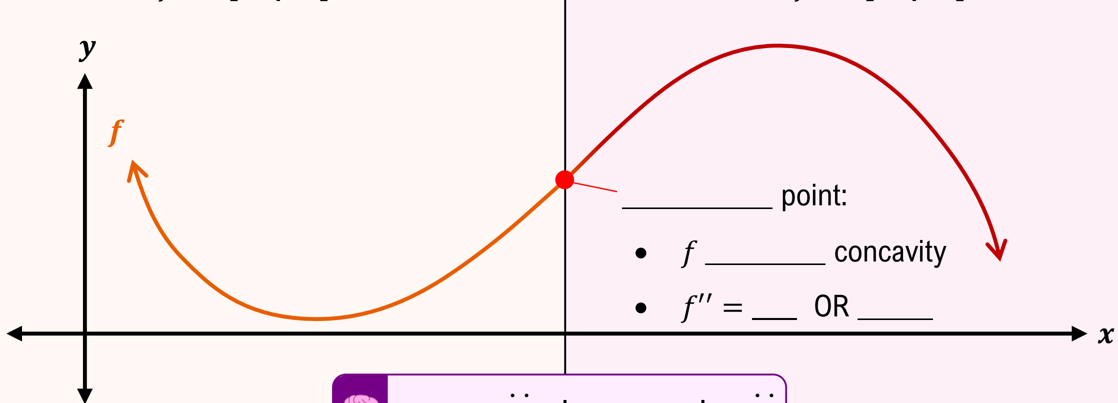
<u>**TOPIC: CONCAVITY**</u>

<u>**Determining Concavity from the Graph of f**</u>

◆ Recall: The sign of f' shows if a function is increasing or decreasing.

► The sign of ____ (__________________ of f'), shows a function's **concavity**, affecting the **shape** of a graph.

Concave Up	Concave Down
f'' is **[+ | −]**	f'' is **[+ | −]**



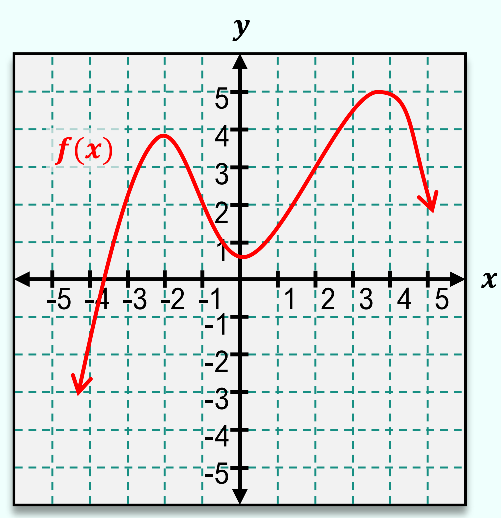
concave $\ddot{\cup}$p | concave dow$\ddot{\cap}$

EXAMPLE Identify the intervals where $f(x)$ is concave up or concave down. State the point(s) of inflection.

Concave **up** (f'' +) on ____________

Concave **down** (f'' −) on ____________

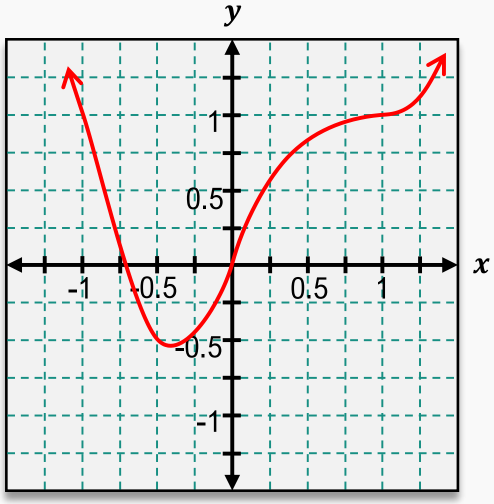
Inflection point(s) (f'' 0 or DNE): ____________



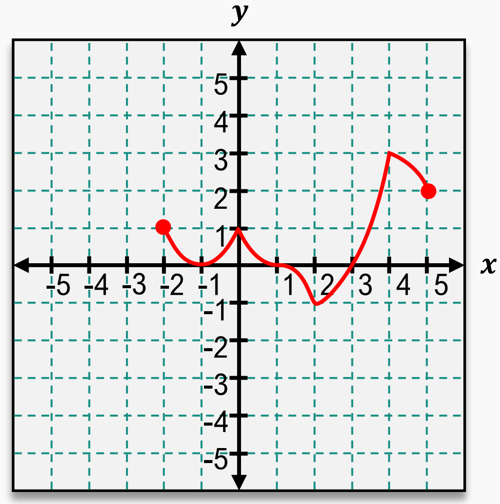
TOPIC: CONCAVITY

PRACTICE For the following graph, find the open intervals for which the function is concave up or concave down. Identify any inflection points.

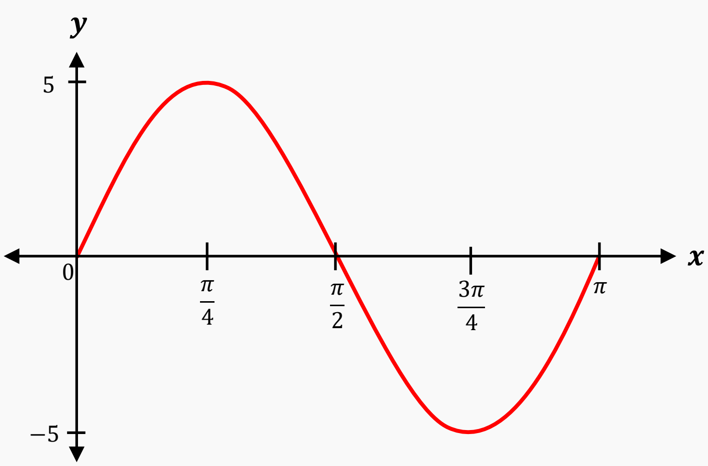
($\boldsymbol{A}$)



($\boldsymbol{B}$)



PRACTICE For the following graph, find the open intervals for which the function is concave up or concave down. Identify any inflection points.

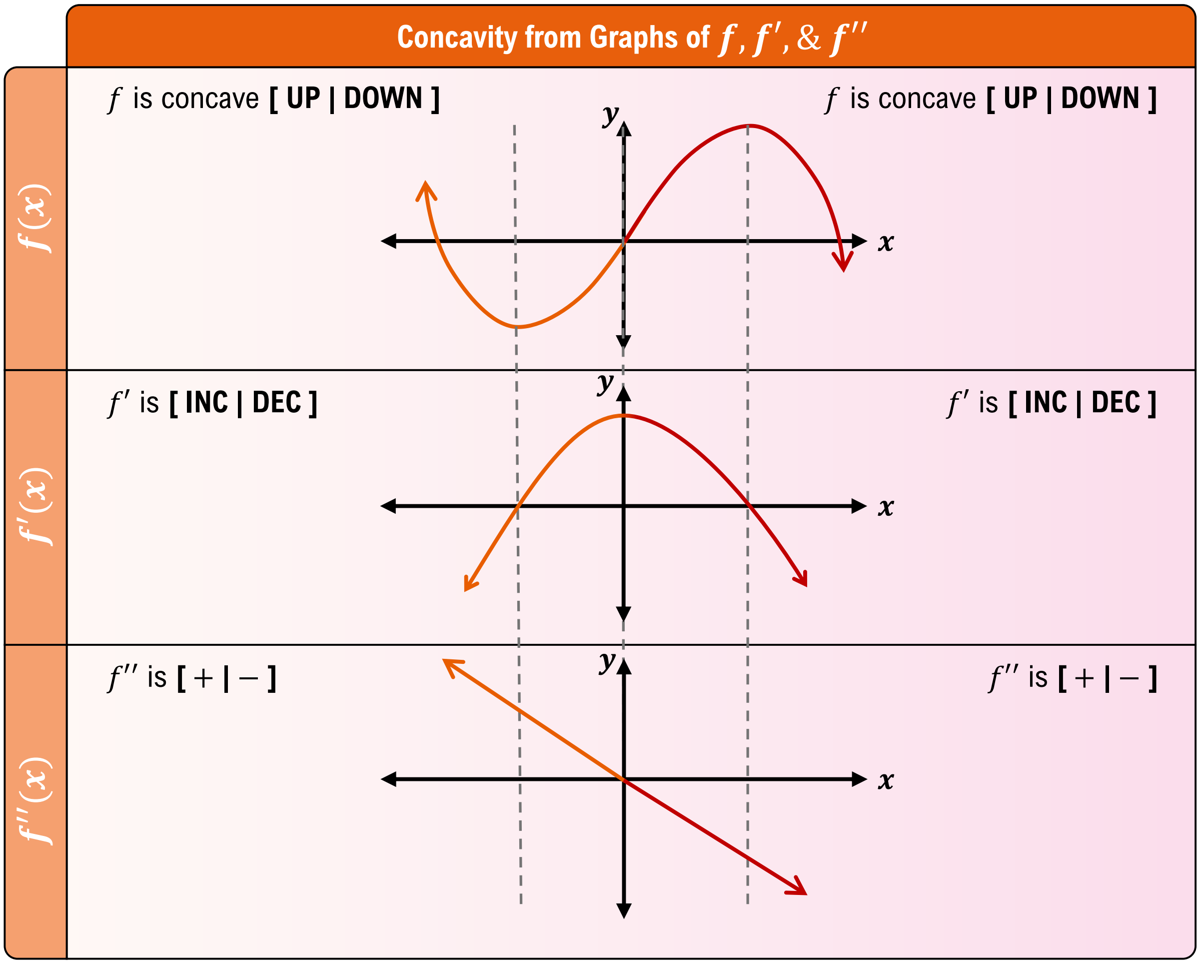


TOPIC: CONCAVITY

Determining Concavity from a Graph of f' or f''

◆ Recall: If f'' is positive, f is concave up. If f'' is negative, f is concave down.

► Since f'' is the rate of change of f', we can also determine concavity by whether f' is _________ or _________.



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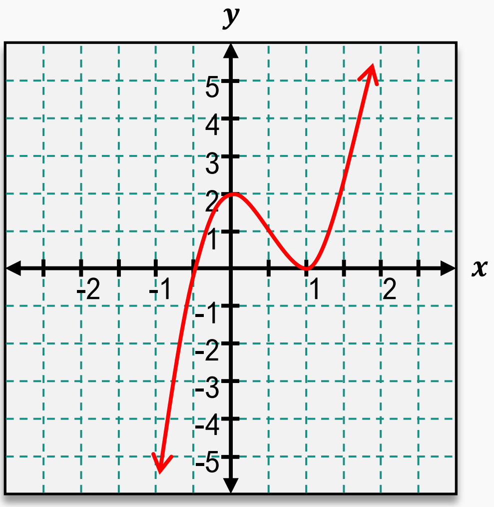
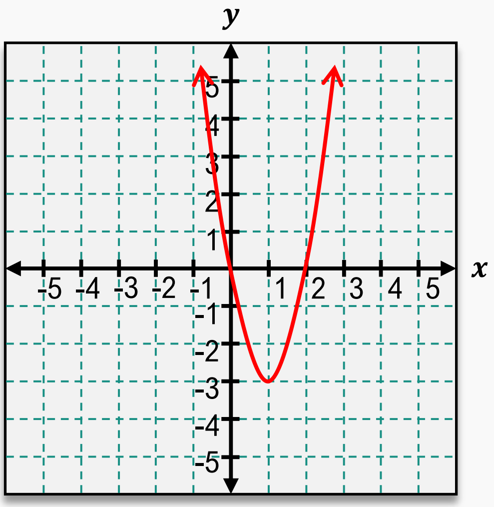
PRACTICE

The graph of $f'(x)$ is shown below. Use the graph to determine the intervals for which $f(x)$ is concave up or concave down and the location of any inflection points.



PRACTICE

The graph of $f''(x)$ is given below. Use the graph to determine the intervals for which $f(x)$ is concave up or concave down and the location of any inflection points.



TOPIC: CONCAVITY

EXAMPLE Use the graph of $f(x)$ below to identify the following:

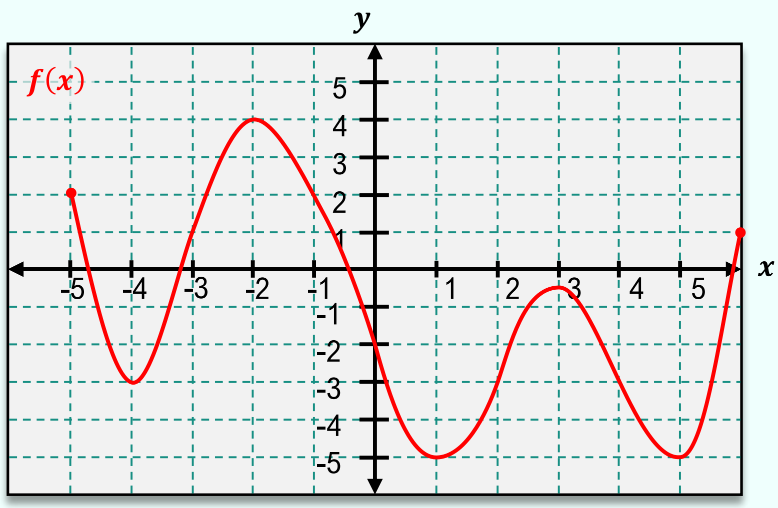
$(\boldsymbol{A})$ Intervals for which $f(x)$ is increasing

$(\boldsymbol{B})$ Intervals for which $f(x)$ is decreasing

$(\boldsymbol{C})$ Intervals for which $f(x)$ is concave up

$(\boldsymbol{D})$ Intervals for which $f(x)$ is concave down

$(\boldsymbol{E})$ Coordinates of the point(s) of inflection



TOPIC: CONCAVITY

Determining Concavity Given a Function

◆ Recall: If f'' is $+$, f is concave up. If $-$, f is concave down. Concavity changes at inflection pts ($f'' = 0$ or DNE).

► Given a fcn, determine concavity by finding potential inflection pts, then testing the sign of f'' __________ those pts.

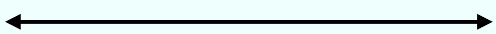
EXAMPLE Determine the intervals for which $f(x)$ is concave up or down.

$f(x) = 2x^3 + 12x^2 + 9x - 4$

Concave **up** on __________

Concave **down** on __________

f'':



HOW TO: Determine Intervals of Concavity

1) Find potential inflection points:

$f''(x) =$ ___ or _____

2) Make sign chart ________ based on potential **inflection points**

3) Plug value from each int. into f'':

If $+$, f concave ___ on int.

If $-$, f concave ______ on int.

TOPIC: CONCAVITY

EXAMPLE Determine the intervals for which the function is concave up or concave down. State the inflection points.

$(\boldsymbol{A})$

$$f(x) = 9x^{\frac{1}{5}}$$

HOW TO: Determine Intervals of Concavity

1) Find potential inflection points:

$f''(x) = 0$ or DNE

2) Make sign chart **intervals** based on potential **inflection points**

3) Plug value from each int. into f'':

If $+$, f concave **UP** on int.

If $-$, f concave **DOWN** on int.

$(\boldsymbol{B})$

$$f(x) = 5x^2 - 2x + 24$$

TOPIC: CONCAVITY

PRACTICE Determine the intervals for which the function is concave up or concave down. State the inflection points.

(A)

$f(x) = 2x(x+9)^2$

(B)

$$f(x) = \frac{7}{2x-9}$$

(C)

$f(x) = 2\sin x + 3x \qquad 0 < x < 2\pi$

HOW TO: Determine Intervals of Concavity

1) Find potential inflection points:

$f''(x) = 0$ or DNE

2) Make sign chart **intervals** based on potential **inflection points**

3) Plug value from each int. into f'':

If $+$, f concave **UP** on int.

If $-$, f concave **DOWN** on int.