TOPIC: DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Derivatives of Inverse Sine & Inverse Cosine Functions

◆ Since $y = \sin^{-1} x$ and ____________ are equivalent, we can use implicit differentiation to find $\frac{d}{dx}\sin^{-1} x$.

New — **Derivative of Inverse Sine**

$y = \sin^{-1} x \quad \Leftrightarrow \quad x = \sin y \qquad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$x = \quad \sin y$

$1 =$ __________

$\frac{dy}{dx} =$ _____ $\longrightarrow$

Recall
$\sin^2 y + \cos^2 y = 1$

$\frac{d}{dx}\sin^{-1} x = \frac{1}{\quad}$, $|x|$ _____

◆ We can use the same process to find the derivative of inverse cosine: $\frac{d}{dx}\cos^{-1} x = \frac{1}{\sqrt{1 - x^2}}, |x| < 1$

EXAMPLE Find the derivative of the following functions.

(A) $f(x) = \sin^{-1}(3x + 2)$

(B) $g(x) = 4\cos^{-1}(6x)$

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PRACTICE Find the derivative of the given function.

(A)

$$f(x) = (x^3 + 4x)\sin^{-1} x$$

(B)

$$y = 4\arccos(3x^6 - x^5)$$

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EXAMPLE Given $4\sin^{-1}x + \cos^{-1}y = \frac{\pi}{3}$, find $\frac{dy}{dx}$ at $\left(0, \frac{1}{2}\right)$.

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Derivatives of Other Inverse Trigonometric Functions

◆ Just like for $\sin^{-1} x$ & $\cos^{-1} x$, we can use implicit differentiation to find a derivative rule for other inverse trig fcns.

► Use *ALL* derivative rules together to find the derivative of more complicated functions.

EXAMPLE Find the derivative of the following functions.

(A) $f(x) = 4x^2 \cdot \tan^{-1} x$

Recall

$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$ $\qquad$ $|x| < 1$

New

$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$ $\qquad$ $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$

$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad$ $\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$ $\qquad$ $|x|$ ____

(B) $g(x) = 6\cot^{-1} x + 4\sec^{-1}(3x+1)$

(C) $h(x) = (\csc^{-1} x)^3$

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PRACTICE Find the derivative of the given function.

$(\boldsymbol{A})$

$f(x) = \tan^{-1}(x^2)$

$(\boldsymbol{B})$

$f(t) = \csc^{-1}(2t + 7)$

$(\boldsymbol{C})$

$f(x) = \cot^{-1}(e^{\cos x})$