TOPIC: PARTIAL FRACTIONS

Partial Fraction Decomposition: Distinct Linear Factors

◆ Recall: We can add/subtract *MULTIPLE* fractions into *ONE* by using a common denominator.

► We can expand *ONE* rational fcn into the sum of *MULTIPLE* simpler fractions using partial fraction decomp.

EXAMPLE Set up the form of the partial fraction decomposition for $f(x) = \frac{2x+14}{x^2+8x+15}$.

New **Partial Fractions – Distinct Linear Factors**

$$f(x) = \frac{P(x)}{Q(x)} = \frac{P(x)}{(\_\_\_\_\_)(\_\_\_\_\_)} = \frac{\_\_\_\_\_}{(\_\_\_\_\_)} + \frac{\_\_\_\_\_}{(\_\_\_\_\_)}$$

$$f(x) = \frac{2x+14}{x^2+8x+15} = \frac{2x+14}{(\_\_\_\_\_)(\_\_\_\_\_)} = \frac{\_\_\_\_\_}{(\_\_\_\_\_)} + \frac{\_\_\_\_\_}{(\_\_\_\_\_)}$$

EXAMPLE Find A & B for the partial fraction decomposition above.

$$\frac{2x+14}{(x+3)(x+5)} = \frac{A}{(x+3)} + \frac{B}{(x+5)}$$

HOW TO: Decompose into Partial Fractions

1) Factor the **denominator** $Q(x)$

2) Set up fraction for each factor of $Q(x)$

- *Linear factors:* use **constant** numerators $(A, B, C, \ldots)$
- *Irreduc. quad. factors:* use **linear** numerators $(Ax + B, \ldots)$

3) Multiply equation by common denom.

4) **Expand** & **group** like terms

5) Set coefficients equal

6) Solve **system of equations**

7) Plug $A, B, C, \ldots$ into fractions from **2)**

◆ If a rational function is improper (degree of $P(x)$ ____ degree of $Q(x)$), then do polynomial division first.

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EXAMPLE Find the partial fraction decomposition using a system of equations.

$$\frac{3x^2 + 3x - 11}{x^2 + 3x - 10}$$

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PRACTICE Express the rational function as a sum or difference of two simpler fractions. Use a system of equations.

$$\frac{1}{(2x+1)(2x-1)}$$

PRACTICE Express the rational function as a sum or difference of three simpler fractions. Use a system of equations.

$$\frac{4x^2-21x-40}{x^3-x^2-20x}$$

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Partial Fraction Decomposition: Distinct Linear Factors with Shortcut

- Recall: We can expand a rational fcn into the sum of multiple simpler fractions using partial fraction decomp.
  - Instead of using a system of equations, we can solve for $A, B, C\ldots$ by making strategic substitutions for x.

EXAMPLE Find the partial fraction decomposition for $f(x)$.

$$f(x) = \frac{2x+14}{x^2+8x+15} = \frac{2x+14}{(x+3)(x+5)} = \frac{A}{(x+3)} + \frac{B}{(x+5)}$$

$$2x + 14 = A(x+5) + B(x+3)$$

let $x =$ ____:

let $x =$ ____:

HOW TO: Decompose into Partial Fractions Using Strategic Sub.

1) Factor the **denominator** $Q(x)$

2) Set up fraction for each factor of $Q(x)$

- *Linear factors:* use **constant** numerators $(A, B, C, \ldots)$
- *Irreduc. quad. factors:* use **linear** numerators $(Ax + B, \ldots)$

3) Multiply equation by common denom.

4) Use strategic substitutions for x that:

- Make a factor become ________ *OR*
- Result in an *easier* system of eqns

5) Plug $A, B, C, \ldots$ into fractions from **2)**

- This method is sometimes referred to as Heaviside's (or Cover-up) method.

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PRACTICE Give the partial fraction decomposition for the following expression using strategic substitutions for x.

$$\frac{4x^2 - 8x - 8}{x(x-2)(x+2)}$$

PRACTICE Express the rational function as a sum or difference of simpler rational expressions.

$$\frac{4}{x^3 - 3x^2 - x + 3}$$

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PRACTICE Express the rational function as a sum or difference of simpler rational expressions.

$$\frac{13x + 2}{2x^2 - x - 1}$$

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Integration Using Partial Fractions

◆ Recall: To decompose into partial fractions, either use a system of equations or strategic substitutions for x.

► To integrate a proper rational function w/ a factorable denominator, first try partial fraction decomposition.

EXAMPLE Evaluate the integral of $f(x) = \frac{2x-5}{x^2-x}$.

$(\boldsymbol{A})$ Find a partial fraction decomposition for $f(x)$.

$(\boldsymbol{B})$ Evaluate $\int f(x)\,dx$.

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EXAMPLE

Determine which method (long division, substitution, completing the square, or partial fraction decomposition) is necessary to solve the integral.

$(\boldsymbol{A})$

$$\int \frac{3}{x^2 - 2x + 17}\ dx$$

$(\boldsymbol{B})$

$$\int \frac{4}{x^2 - 2x - 15}\ dx$$

$(\boldsymbol{C})$

$$\int \frac{2x - 2}{x^2 - 2x - 15}\ dx$$

$(\boldsymbol{D})$

$$\int \frac{2x^3 - 4x^2 - 29x - 1}{x^2 - 2x - 15}$$

$(\boldsymbol{E})$

$$\int \frac{3x^3 - 6x^2 - 45x + 4}{x^2 - 2x - 15}$$

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PRACTICE Evaluate the integral.

(***A***)

$$\int \frac{-6x^2 + 3x + 5}{x^3 - x}\, dx$$

(***B***)

$$\int_0^3 \frac{3x + 10}{x^2 + 9x + 20}\, dx$$

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PRACTICE Evaluate the integral.

$$\int \frac{4x^2 + 2x + 3}{x^2 + x}\, dx$$

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PRACTICE Find the area under the curve of $y = \frac{10}{(x+2)(x+3)}$ between $x = 0$ and $x = 4$.

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EXAMPLE Use substitution to find the integral.

$$\int \frac{\cos x}{\sin x\,(\sin x - 1)}\,dx$$

Hint: let $u = sin\,x$

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Partial Fraction Decomposition: Repeated Linear Factors

◆ Recall: To decompose into partial fractions, either use a system of equations or strategic substitutions for x.

► When a linear factor is repeated (raised to an exponent n), add a fraction for each power from 1 to ____.

New

$$f(x) = \frac{P(x)}{Q(x)} = \frac{P(x)}{(\_\_\_\_\_\_)^n} = \frac{\_\_\_\_\_\_}{(\_\_\_\_\_\_)} + \frac{\_\_\_\_\_\_}{(\_\_\_\_\_\_)^2} + \cdots + \frac{\_\_\_\_\_\_}{(\_\_\_\_\_\_)^n}$$

(Partial Fraction Decomposition: Repeated Linear Factors)

EXAMPLE Find the partial fraction decomposition of $\frac{2x^2-5x+1}{(x+1)(x-1)^2}$.

$$\frac{2x^2 - 5x + 1}{(x+1)(x-1)^2} = \frac{\_\_\_\_\_\_}{(\_\_\_\_\_\_)} + \frac{\_\_\_\_\_\_}{(\_\_\_\_\_\_)} + \frac{\_\_\_\_\_\_}{(\_\_\_\_\_\_)^2}$$

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EXAMPLE Use the method of partial fractions to evaluate the integral.

$$\int_1^4 \frac{x-1}{x^2(x+1)}\,dx$$

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PRACTICE Express the integrand as a sum of partial fractions and evaluate the integral.

$$\int \frac{2}{x^3 + x^2 - x - 1}\, dx$$

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EXAMPLE

Find the volume of the solid formed by revolving the region bounded by $f(x)$, $x = 2$, $x = 4$, and the x-axis about the x-axis.

Recall

$$Volume = \int_a^b A(x)dx$$

$$y = \frac{1}{x\sqrt{x-1}}$$

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Partial Fraction Decomposition: Irreducible Quadratic Factors

- If the denominator of a rational fcn has irreducible quadratic factors, numerators of decomposition will be linear.
  - Like *repeated* linear factors, for *repeated* irreducible quad factors, add fractions for each power from 1 to n.

New

$$f(x) = \frac{P(x)}{Q(x)} = \frac{P(x)}{(ax^2+bx+c)^n} = \frac{\_\_\_\_\_\_\_\_}{(\_\_\_\_\_\_\_\_)} + \frac{\_\_\_\_\_\_\_\_}{(\_\_\_\_\_\_\_\_)} + \cdots + \frac{\_\_\_\_\_\_\_\_}{(\_\_\_\_\_\_\_\_)}$$

(Partial Fraction Decomposition: Irreducible Quadratic Factors)

EXAMPLE Find the partial fraction decomposition of $\frac{4x^2-3x+5}{(x-2)(x^2+1)}$.

$$\frac{4x^2-3x+5}{(x-2)(x^2+1)} = \frac{\_\_\_\_\_\_\_\_}{(\_\_\_\_\_\_\_\_)} + \frac{\_\_\_\_\_\_\_\_}{(\_\_\_\_\_\_\_\_)}$$

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EXAMPLE Evaluate the integral.

$$\int_1^2 \frac{x-5}{(x+1)(x^2+3x-1)}\,dx$$

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EXAMPLE Use the method of partial fractions to evaluate the integral.

$$\int \frac{5x^2 + 6}{(x^2 + 1)^2}\,dx$$

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PRACTICE Use the method of partial fractions to evaluate the integral.

$$\int \frac{5x^3 - x^2 + 7x - 2}{(x^2+1)(x^2+2)}\,dx$$

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PRACTICE Evaluate the integral.

$$\int \frac{t^4 + t^2 - 2}{t^3 + t}\, dt$$