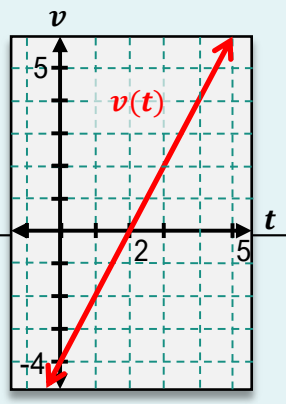
TOPIC: KINEMATICS

Motion Problems With Integrals: Velocity & Position

◆ Recall: Velocity is the derivative of position $v(t) = ds/dt$ & displacement is change in position $\Delta s = s(t_f) - s(t_i)$.

EXAMPLE Suppose that an object moves in a line such that its velocity is $v(t) = 2t - 4$.

Name	Formula	Example
Displacement (Δs) on an Interval $[t_i, t_f]$	$\Delta s = s(t_f) - s(t_i) = \int_{t_i}^{t_f} \_\_\_\_ dt$	Find the displacement of the object on the interval $[0, 5]$. $\Delta s = \int_{\_\_}^{\overline{\_\_}} (\_\_\_\_\_)\, dt$
Total Distance on an Interval $[t_i, t_f]$	$\int_{t_i}^{t_f} v(t)\ dt$	Find the total distance of the object on the interval $[0, 5]$. $Total\ Dist. = \int_{\_\_}^{\overline{\_\_}} \lvert\_\_\_\_\_\rvert dt$
Position Function ($s(t)$)	$s(t) = \_\_\_\_ + \int_{t_i}^{\overline{\_\_}} v(\_\_)\, d\_\_$	Find the position function given that $s(0) = 3$. $s(t) = \_\_\_\_ + \int_{\_\_}^{\overline{\_\_}} \_\_\_\_\_$



◆ To find a future position, plug $t = t_f$ into position function $s(t)$.

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PRACTICE

The velocity (mi/hr) of a drone flying in the air is given by $v(t) = 12 + 4t^2$ for $0 \leq t \leq 4\ hours$. Let $s(0) = 0$.

$(\boldsymbol{A})$ Determine $s(t)$ for $0 \leq t \leq 4$.

$(\boldsymbol{B})$ How far does the drone travel during the first hour?

$(\boldsymbol{C})$ How far has the drone traveled by the time it has reached 48 mi/hr?

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PRACTICE

Suppose that a particle travels along the x-axis and its velocity is given by $v(t) = 2\cos t$ for $0 \leq t \leq 2\pi$.

$(\boldsymbol{A})$ Find the particle's displacement on $\left[0, \frac{5\pi}{4}\right]$.

$(\boldsymbol{B})$ Find the total distance traveled by the particle on $\left[0, \frac{5\pi}{4}\right]$.

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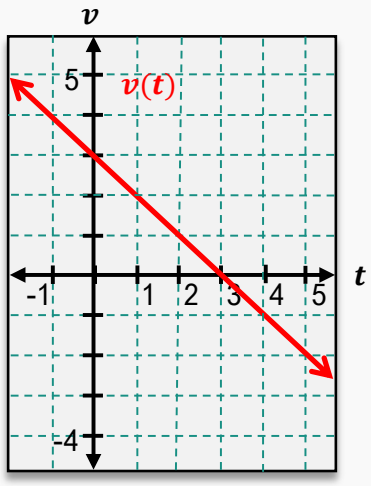
PRACTICE A particle travels in a straight line and its velocity is given on the graph as $v(t)$.

$(\boldsymbol{A})$ Find displacement on $[0, 5]$.



$(\boldsymbol{B})$ Find total distance on $[0, 5]$.

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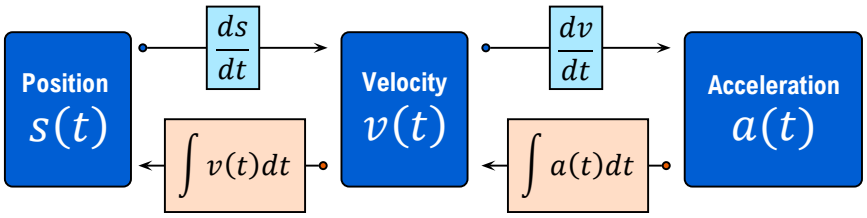
Motion Problems With Integrals: Acceleration

◆ Recall: Before, we integrated velocity to find position. Now, we can integrate acceleration to find velocity.



EXAMPLE The acceleration of a particle moving along the x-axis is $a(t) = 6t - 12\ m/s^2$.

$(\boldsymbol{A})$ Find the velocity function, given that $v(0) = -9\ m/s$.

New

$$v(t) = v(t_i) + \int_{t_i}^{t} a(x)dx$$

$(\boldsymbol{B})$ Find the position function, given that $s(0) = 1\ m$.

Recall

$$s(t) = s(t_i) + \int_{t_i}^{t} v(x)dx$$

$(\boldsymbol{C})$ What is the position of the particle at time $t = 8\ s$?

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PRACTICE

At $t = 0$, a car approaching a stop sign decelerates from a speed of $50\ mi/hr$ according to the acceleration function $a(t) = 4t + 3$, where $t \geq 0$ and is measured in hours. How far does the car travel between $t = 0$ and $t = 0.1\ hr$?

PRACTICE A particle moves along the x-axis and its acceleration is given by $a(t) = \cos \pi t$.

($\boldsymbol{A}$) Find $v(t)$ if $v(0) = 0$.

($\boldsymbol{B}$) Find $s(t)$ if $s(0) = 1$.

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PRACTICE

A rock is thrown from a height of $2\ ft$ with an initial speed of $25\ ft/s$. Acceleration resulting from gravity is $-32\ ft/s^2$.

$(\boldsymbol{A})$ Find $v(t)$.

$(\boldsymbol{B})$ Find $s(t)$.

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EXAMPLE A particle moves so that its acceleration (m/s^2) is defined by $a(t) = 2t - 1$ where $t \geq 0\ seconds$.

$(\boldsymbol{A})$ Find $v(t)$ if $v(0) = 6\ m/s$.

$(\boldsymbol{B})$ Find displacement of the particle during the first 5 seconds.

$(\boldsymbol{C})$ Find total distance traveled by the particle during the first 5 seconds.