TOPIC: ESTIMATING AREA WITH FINITE SUMS

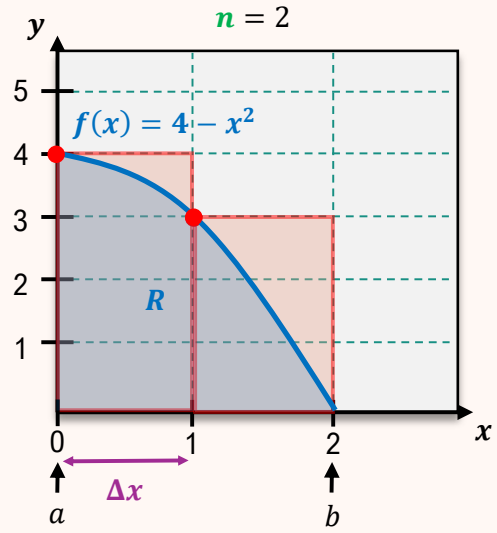
Estimating the Area Under a Curve Using Left Endpoints

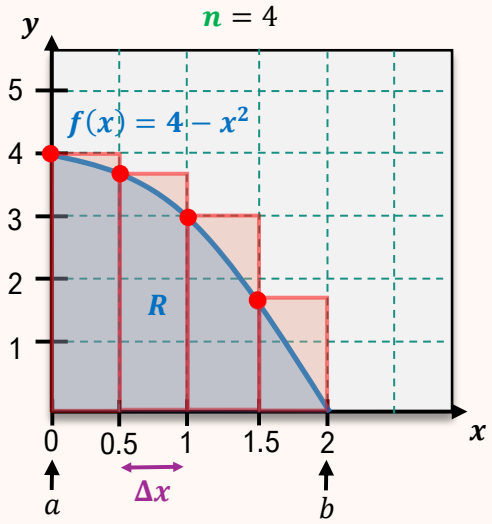
◆ To estimate the area under a curve, break it into many (n) _____ and add all of their areas.

► The height of each rectangle is the *function value* at the left endpoint and the width is $\Delta x = \frac{b-a}{n}$.

EXAMPLE Approximate the area under the curve (region R) using (A) 2 rectangles and (B) 4 rectangles.

New **Estimating Area Under A Curve Using Left Endpoints**





$A \approx$

$\approx$

$\approx \quad \cdot \quad + \quad \cdot$

$\approx$

$\approx$

$A \approx$

$\approx$

$\approx \quad (\quad + \quad + \quad + \quad)$

$\approx$

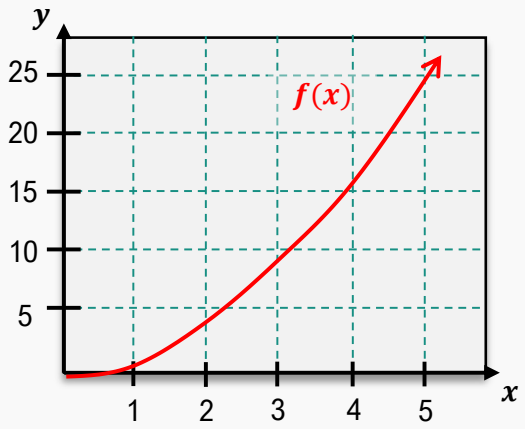
$\approx$

◆ The more rectangles (*subintervals*) we break our region into, the more _____ our estimate gets.

TOPIC: ESTIMATING AREA WITH FINITE SUMS

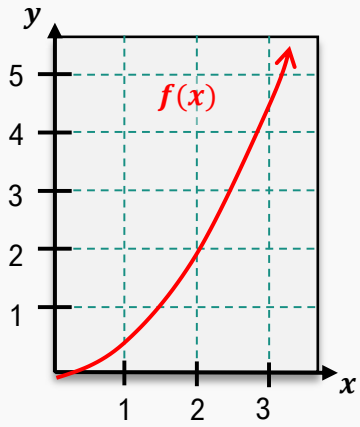
PRACTICE

Use five rectangles to estimate the area under the curve of $f(x) = x^2$ from $x = 0$ to $x = 5$ using left endpoints.



PRACTICE

Use two rectangles to estimate the area under the curve of $f(x) = \frac{1}{2}x^2$ from $x = 0$ to $x = 3$ using left endpoints.



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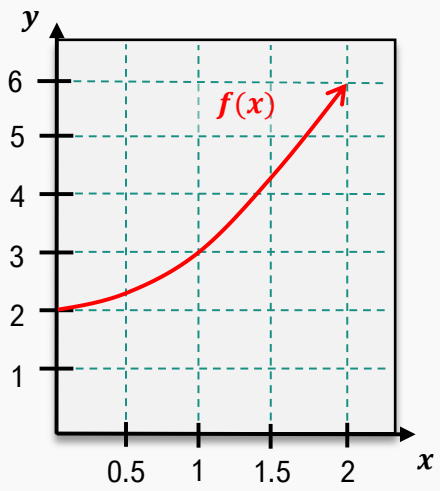
PRACTICE

Use four rectangles to estimate the area under the curve of $f(x) = x^2 + 2$ from $x = 0$ to $x = 2$ using left endpoints.



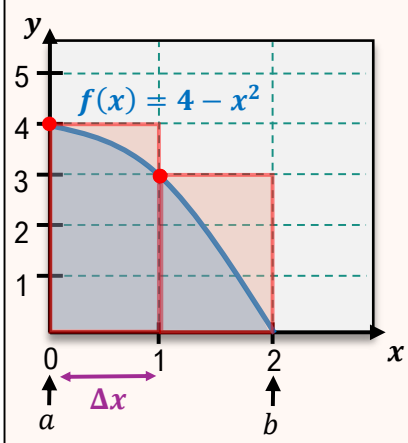
TOPIC: ESTIMATING AREA WITH FINITE SUMS

Estimating the Area Under a Curve Using Right Endpoints & Midpoints

◆ Recall: We can estimate the area under a curve by breaking it into rectangles (subintervals) and using left endpts.

► We can also use the fcn value at the _____ endpoint or _____ as the height of each rectangle.

Recall **Left Endpt Approx.**



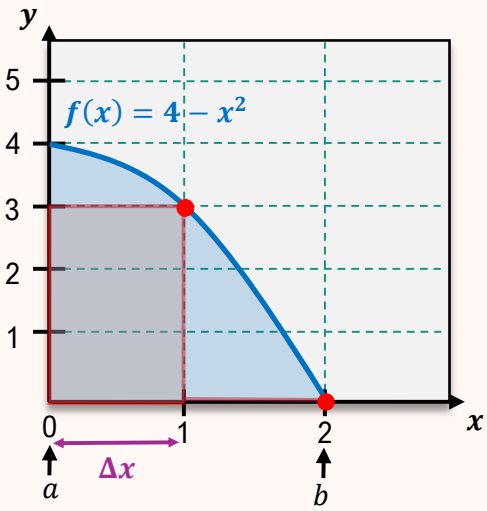
$$\Delta x = \frac{b-a}{n}$$

$A \approx A_1 + A_2$

$\approx w_1 \cdot h_1 + w_2 \cdot h_2$

$\approx \Delta x \cdot f(0) + \Delta x \cdot f(1)$

New **Right Endpt Approx.**

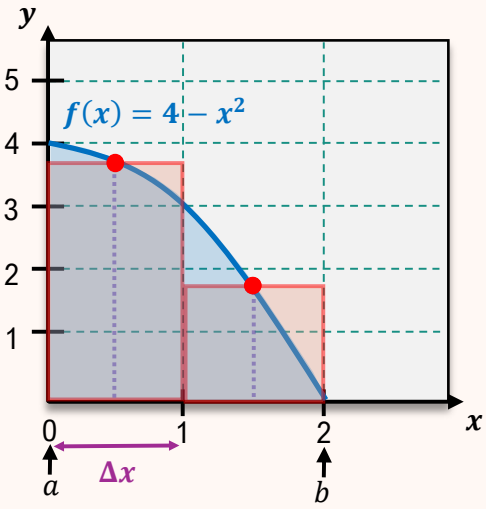


$A \approx A_1 + A_2$

$\approx w_1 \cdot h_1 + w_2 \cdot h_2$

$\approx \Delta x \cdot \quad + \Delta x \cdot$

New **Midpoint Approx.**



$A \approx A_1 + A_2$

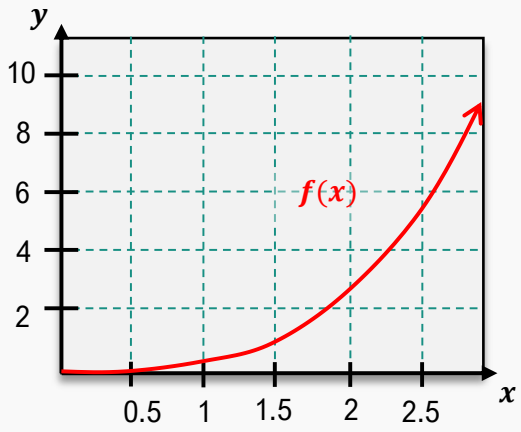
$\approx w_1 \cdot h_1 + w_2 \cdot h_2$

$\approx \Delta x \cdot \quad + \Delta x \cdot$

TOPIC: ESTIMATING AREA WITH FINITE SUMS

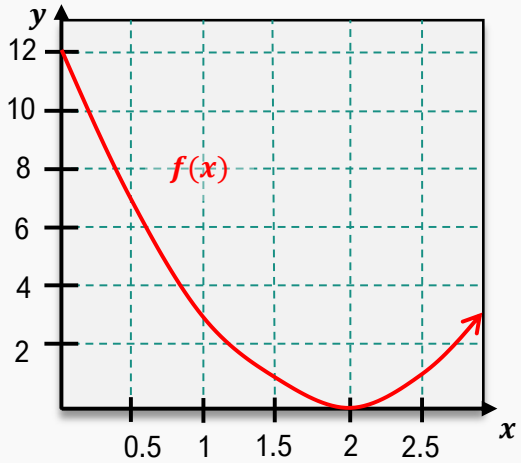
PRACTICE

Use three rectangles to estimate the area under the curve of $f(x) = \frac{1}{3}x^3$ from $x = 0$ to $x = 3$ using the right endpoints.



PRACTICE

Use three rectangles to approximate the area under the curve of $f(x) = 3(x-2)^2$ from $x = 0$ to $x = 3$ using the midpoint rule.



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EXAMPLE

Approximate the area under the curve of $f(x) = x^3$ on the interval $[0, 3]$ using 3 rectangles for all the following methods:

a. Left Endpoints

b. Right Endpoints

c. Midpoint rule