TOPIC: EULER'S METHOD

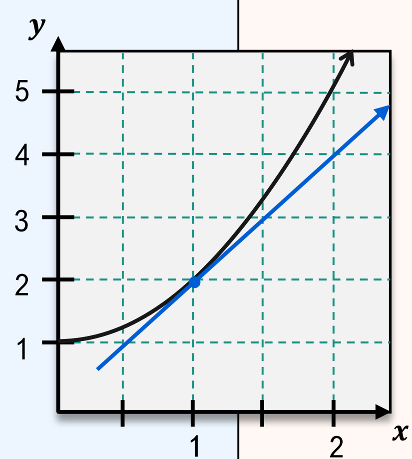
Euler's Method

◆ Recall: Approximate $f(x)$ w/ a *single* tangent line using the linearization $L(x) = f(a) + f'(a)(x - a)$.

► Approx. $f(x)$ more accurately using Euler's method w/ *multiple* __________ tangent lines that "chase the curve."

EXAMPLE Given $y' = 2x$ and the initial condition $(x_0, y_0) = (1, 2)$, approximate $f(2)$ using Euler's method with two steps of size $h = 0.5$.

Recall — **Linear Approximation**



$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

$$f(2) \approx L(2) = f(1) + 2(1)(2 - 1)$$
$$= 2 + 2(1)$$
$$= 4$$

New — **Euler's Method**

$$x_n = x_{n-1} + h$$

"______" value "______" value ______ size

$$y_n = y_{n-1} + h \cdot f'(x_{n-1}, y_{n-1})$$

__________ at "____" pt

n	0	1	2
x_n	$x_0 =$ __		
y_n	$y_0 =$ __		

◆ Euler's method is more accurate with __________ step sizes of h.

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EXAMPLE

Estimate the following solution using Euler's Method with $n = 5$ steps over the interval $[0,1]$. Then compare your approximated solution at $x = 1$ to the exact solution given.
Hint: Divide your interval by the number of steps to get the step size.

$y' = 2x - y,\ y(0) = -1$ Exact Solution: $y = 2x + e^{-x} - 2$

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PRACTICE

Use Euler's method with a step size of $h = 0.5$ to estimate the value of $y(2)$, where y is the solution of the initial value problem $y' = 2x$, $y(0) = 1$.

PRACTICE

Let $y'(t) = \frac{y}{2}$ with $y(0) = 2$. Compute the first three approximations given by Euler's Method with a step size of $h = 0.2$.