TOPIC: INTEGRALS OF EXPONENTIAL FUNCTIONS

Integrals of General Exponential Functions

◆ Recall: To find the *derivative* of a general exponential function b^x, multiply b^x by $\ln b$.

► To find the *integral* of a general exponential function b^x, _____ b^x by $\ln b$.

Recall: Rule for *Derivative* of b^x	New: Rule for *Integral* of b^x
$\frac{d}{dx}(b^x) = b^x \cdot \ln b$	$\int b^x \, dx = \text{——} + C$ $\quad b > 0,\ b \neq 1$
$\frac{d}{dx}(7^x) = 7^x \cdot \ln 7$	$\int 7^x \, dx =$

EXAMPLE Evaluate the indefinite integral.

$$\int \left(3\left(\frac{1}{2}\right)^x + 8^x\right) dx$$

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PRACTICE Evaluate the indefinite integrals.

Recall

$$\int b^x \, dx = \frac{b^x}{\ln b} + C$$

(A)

$$\int -(6)^x \, dx$$

(B)

$$\int 3x^4 - (5)^x \, dx$$

(C)

$$\int 2\left(\frac{1}{3}\right)^x \, dx$$

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EXAMPLE Evaluate the integrals using substitution.

New

$$\int b^u \, du = \frac{b^u}{\ln b} + C$$

(A)

$$\int 4^{7x} \, dx$$

(B)

$$\int 5^{\cos\theta} \sin\theta \; d\theta$$

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PRACTICE Evaluate the definite integral.

$$\int_0^5 \frac{3^{\sqrt{x+4}}}{\sqrt{x+4}}\,dx$$

Recall

$$\int b^u\,du = \frac{b^u}{\ln b} + C$$

PRACTICE Find the area under the graph of $f(x) = 3^{2x}$ between $x = 0$ and $x = 2$.

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Integrals of Natural Exponential Functions (e^x)

◆ Recall: $y = e^x$ is just a special case of $y = b^x$ where $b = e$.

► We can use the integral rule for general exponential functions to find the integral of e^x.

RULES OF INTEGRATION		
Name	**Rule**	**Example**
General Exponential	$\int b^x \, dx = \frac{b^x}{\ln b} + C$ $\quad b > 0,\ b \neq 1$	$\int 7^x \, dx = \frac{7^x}{\ln 7} + C$
Natural Exponential	$\int e^x \, dx = \text{——} + C = \text{\_\_\_\_\_} + C$	$\int 5e^x \, dx =$

EXAMPLE Find the indefinite integral.

$$\int (3x^4 - \pi e^x + 2) \, dx$$

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PRACTICE Evaluate the indefinite integrals.

(***A***)

$$\int (\sqrt{x} - e^x)\, dx$$

(***B***)

$$\int \left(4e^x + \frac{1}{x^3}\right) dx$$

EXAMPLE Evaluate the indefinite integral.

$$\int x^2 e^{x^3}\, dx$$

New

$$\int e^u du = e^u + C$$

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PRACTICE Evaluate the indefinite integrals.

Recall

$$\int e^u du = e^u + C$$

(A)

$$\int \frac{e^{2\sqrt{x}}}{\sqrt{x}}\, dx$$

(B)

$$\int e^{\sec\theta} \sec\theta \tan\theta \; d\theta$$

EXAMPLE Evaluate the integrals using substitution.

(A)

$$\int \frac{e^t + e^{4t}}{e^{2t}} dt$$

(B)

$$\int 3e^{3x}(1 + e^{3x})^5 dx$$

TOPIC: INTEGRALS OF EXPONENTIAL FUNCTIONS

PRACTICE Find the definite integral.

$$\int_{-1}^{1} x^2 e^{x^3}\, dx$$

PRACTICE Find the area of $f(x) = 3 + e^{2x}$ from $x = 0$ to $x = 2$.