TOPIC: BASICS OF DIFFERENTIAL EQUATIONS

Classifying Differential Equations

◆ A differential equation (DE) is an equation involving a function $y = f(x)$ and its ________________.

► A DE can be classified by its order (___________ derivative) and whether it is linear or nonlinear.

EXAMPLE Determine the order of the following differential equations and indicate if they are linear.

(A)

$$y'' + 4y' - 3y = \sin(e^x)$$

Order: ___

Dependent var. (often y) & its derivatives are…

Not multiplied by each other ☐

Only raised to 1^{st} power ☐

Not the "inside" of a function ☐

[LINEAR | NONLINEAR]

(B)

$$ty'' - t^2y^{(4)} = e^y$$

Order: ___

Dependent var. (often y) & its derivatives are…

Not multiplied by each other ☐

Only raised to 1^{st} power ☐

Not the "inside" of a function ☐

[LINEAR | NONLINEAR]

(C)

$$y \cdot \frac{dy}{dx} + 5\left(\frac{dy}{dx}\right)^3 - 4 = 2x$$

Order: ___

Dependent var. (often y) & its derivatives are…

Not multiplied by each other ☐

Only raised to 1^{st} power ☐

Not the "inside" of a function ☐

[LINEAR | NONLINEAR]

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PRACTICE State the order of the differential equation and indicate if it is linear or nonlinear.

$(\boldsymbol{A})$

$$y''' + 3xy = 4\sqrt{x}$$

$(\boldsymbol{B})$

$$\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)(1 - x) = 2$$

$(\boldsymbol{C})$

$$(y'')^2 + 6e^t y' = 4t$$

$(\boldsymbol{D})$

$$y' \cdot y = 3e^t$$

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Verifying Solutions of Differential Equations

◆ The solution of a DE is any function $y = f(x)$ that makes the equation true when y & its *derivatives* are plugged in.

EXAMPLE Verify that the following functions are solutions to the given differential equations.

$(\boldsymbol{A})$

$y = e^{2x};\ 3y' - 5y = e^{2x}$

$(\boldsymbol{B})$

$y = 4 + \ln x;\ \ xy'' + y' = 0$

◆ DEs can also be in *implicit* form (y is not isolated) which requires *implicit differentiation* of the solution to verify.

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EXAMPLE Use implicit differentiation to show that the given solution satisfies the indicated differential equation.

Solution: $4x^2 = 2y^3 + 4y$; DE: $(3y^2 + 2)y' = 4x$

EXAMPLE Verify that the given function $y(t) = -2\cos 2t$ is a solution of the initial value problem $y'' + 4y = 0,\ y'(0) = 0.$

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EXAMPLE Determine if $y = e^{4x}$ is a solution to the differential equation $y'' - 4y' + 3y = 0$.

EXAMPLE Show that the given function is the general solution of the indicated differential equation. Assume that C is an arbitrary constant.

Solution: $y = \frac{C}{x^2}$

DE: $xy' = -2y$

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EXAMPLE

Verify that the given function is a solution of the differential equation that follows it. Assume C is an arbitrary constant.

$y(t) = Ce^{-6t};\ y'(t) + 6y(t) = 0$

EXAMPLE

Use implicit differentiation to show that the given solution satisfies the indicated differential equation. Assume C is an arbitrary constant.

Solution: $y + x^2y^2 - 3x = C$

DE: $(1 + 2x^2y)y' = 3 - 2xy^2$

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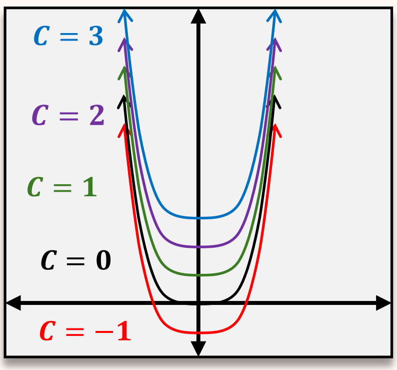
Solutions to Basic Differential Equations

◆ The *general* solution of a basic DE is the antideriv. w/ integration constant C, which represents a ___________ of fcns.

► The *particular* solution of a DE is found by using an initial condition (a _________ on the function curve).

EXAMPLE Find the particular solution to the differential equation $y' = 4x^3$ passing through the point $(1,4)$.

New **Particular Solution of a Differential Equation**



$= \int \qquad dx$

$y =$ **Find general solution**

$= \qquad + C$ **Apply initial condition**

$C =$

$y =$ **Write particular solution**

EXAMPLE Verify that $y = Ce^{x^2}$ is a solution to the differential equation $y' = 2xy$ and find the particular solution that passes through the point $(0,6)$.

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PRACTICE Find the particular solution to the differential equation $y' = 2e^t + 4t$ given the initial condition $y(0) = 1$.

PRACTICE Find the general solution to the differential equation $\frac{dy}{dx} = -2x + 5x^2$.

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PRACTICE

Find the particular solution to the differential equation $y' = 2\sin x + 3\cos x$ given the initial condition $y(0) = 4$.