TOPIC: THE FUNDAMENTAL THEOREM OF CALCULUS

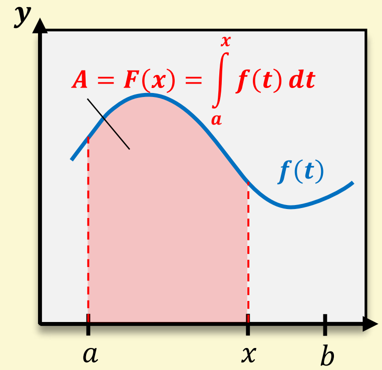
Fundamental Theorem of Calculus (FTC) Part 1

- Recall: $F(x)$ is an antiderivative of $f(x)$.
  - Fundamental Theorem of Calculus Part 1 connects derivatives to definite integrals.

The Fundamental Theorem of Calculus Part 1

If $f(x)$ is continuous on $[a, b]$, and $F(x) = \int_a^x f(t)\,dt$, then:

$$F'(x) = \_\_\_\_ \int_a^x f(t)\,dt = \_\_\_\_\_$$



- If upper bound is a *fcn* of x, use FTC w/ __________ rule to find the derivative.

New

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f\big(g(x)\big)\cdot g'(x)$$

EXAMPLE Use the Fundamental Theorem to find $\frac{dy}{dx}$ for the following.

(A)

$$y = \int_1^x (t^3 - 3t^2 + 4)\,dt$$

(B)

$$y = \int_5^{x^2} \frac{2}{1+t}\,dt$$

TOPIC: THE FUNDAMENTAL THEOREM OF CALCULUS

PRACTICE Given the definite integral $F(x) = \int_3^x [t^8 - \sin(t^4)]dt$, find the derivative $F'(x)$.

PRACTICE Given the definite integral $F(x) = \int_{12}^{20x} \left(h^4 + \frac{63h}{\sqrt{h^5}}\right) dh$, find the derivative $F'(x)$.

TOPIC: THE FUNDAMENTAL THEOREM OF CALCULUS

EXAMPLE Find the derivative $F'(x)$ of the following definite integrals.

$(\boldsymbol{A})$

$$F(x) = \int_0^x \left(t^4 - 30t^3 + \frac{1}{3}t^2 + 30t - 1000\right) dt$$

$(\boldsymbol{B})$

$$F(x) = \int_0^{x^2-13x} \frac{(h^8)}{\sqrt{h} - \sin(h)} dh$$

TOPIC: THE FUNDAMENTAL THEOREM OF CALCULUS

Fundamental Theorem of Calculus (FTC) Part 2

- Recall: The Fundamental Theorem Part 1 gives a relationship between definite integrals and antiderivatives.
  - FTC Part 2 allows us to evaluate a definite integral using the antiderivative at the upper and lower bounds.

The Fundamental Theorem of Calculus Part 2

If $f(x)$ is continuous on $[a, b]$, and $F(x)$ is *any* antiderivative of $f(x)$ on $[a, b]$, then:

$$\int_a^b f(x)\,dx = \_\_\_\_\_\_ - \_\_\_\_\_\_$$

EXAMPLE Evaluate the following integrals.

$(\boldsymbol{A})$

$$y = \int_2^5 2x\,dx$$

$(\boldsymbol{B})$

$$y = \int_1^4 (x^2 - 4x + 5)\,dx$$

TOPIC: THE FUNDAMENTAL THEOREM OF CALCULUS

PRACTICE Evaluate the following integrals:

(A)

$$\int_0^3 4dx$$

(B)

$$\int_1^4 (2x + 1)\, dx$$

(C)

$$\int_{-1}^2 (x^2 - 3x + 2)\, dx$$

TOPIC: THE FUNDAMENTAL THEOREM OF CALCULUS

PRACTICE Evaluate the following integrals:

$(\boldsymbol{D})$

$$\int_0^1 (2x^3 - x^2 + 4x)\, dx$$

$(\boldsymbol{E})$

$$\int_2^3 x^{\frac{5}{2}}\, dx$$

$(\boldsymbol{F})$

$$\int_1^2 \frac{5}{x^2}\, dx$$

TOPIC: THE FUNDAMENTAL THEOREM OF CALCULUS

EXAMPLE Evaluate the following definite integrals:

$(\boldsymbol{A})$

$$\int_{0}^{5} (x^3 + 3x^2 - 6x + 2)dx$$

$(\boldsymbol{B})$

$$\int_{-2}^{4} \frac{13}{y^3} dy$$

$(\boldsymbol{C})$

$$\int_{0}^{\frac{\pi}{2}} \left[\frac{3}{\pi} - \cos(\theta)\right] d\theta$$