Topic Resource: Basic Integration Rules

RULES OF INTEGRATION								
Basic Integration Properties								
$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$		$\int [k \cdot f(x)] dx = k \cdot \int f(x) dx$						
Basic Integration Rules								
$\int 0 dx = C$		$\int k dx = kx + C$						
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \ n \neq -1$		$\int f(g(x)) \cdot g'(x) \ dx = \int f(u) \ du$						
Integrals Resulting in Trig Functions								
$\int \cos x dx = \sin x + C$	$\int \sec^2 x \ dx$	$= \tan x + C$	$\int \sec x \tan x dx = \sec x + C$					
$\int \sin x dx = -\cos x + C$	$\int \csc^2 x \ dx =$	$=-\cot x+C$	$\int \csc x \cot x dx = -\csc x + C$					
Integrals of Other Trig Functions								
$\int \tan x \ dx = -\ln \cos x + C = \ln \sec x + C$		$\int \cot x dx = \ln \sin x + C$						
$\int \sec x dx = \ln \sec x + \tan x + C$		$\int \csc x dx = -\ln \csc x + \cot x$						
Integrals Involving Exponential & Logarithmic Functions								
$\int b^x dx = \frac{b^x}{\ln b} + C$	$\int e^x dx = e^x + C$		$\int \frac{1}{x} dx = \ln x + C$					
$\int b^u du = \frac{b^u}{\ln u} + C$	$\int e^u du = e^u + C$		$\int \frac{1}{u} du = \ln u + C$					
Integrals Resulting in Inverse Trig Functions								
$\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + C$	$\int \frac{1}{a^2 + u^2} du = \frac{u}{a} \tan^{-1} \frac{u}{a} + C$		$\int \frac{1}{ u \sqrt{u^2 - a^2}} du = \frac{1}{a} \sec^{-1} \left \frac{u}{a} \right + C$					
Properties of Definite Integrals								
$\int_{b}^{a} f(x) dx = -\int_{a}^{b} f(x) dx$	$\int_{a}^{a} f(x) dx = 0$		$\left \int_{a}^{c} f(x) dx + \int_{c}^{b} f(x) dx \right = \int_{a}^{b} f(x) dx$					

Intro to Integration by Parts

- ◆ To integrate the product of two functions, if all other methods fail, use integration by parts (IBP).
 - ► To get the IBP formula, we have to _____ the derivative product rule.

New Integration by Parts

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\dots = \int \dots dx + \int \dots dx$$

$$\int \dots dx = \dots - \int \dots dx$$

$$\int u \cdot dv = \dots - \int \dots dx$$

ullet When using IBP, u should become simpler when differentiated and dv should be a function that's easily integrated.

EXAMPLE

Find the indefinite integral using integration by parts.

$$\int 6xe^x dx$$

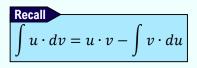
$$u = \underline{\qquad} dv = \underline{\qquad} dx$$

$$du = \underline{\hspace{1cm}} dx \qquad v = \underline{\hspace{1cm}}$$

EXAMPLE

Find the integral.

$$(A) \int \ln x \, dx$$





$$\int \sin^{-1} t \, dt$$

PRACTICE

Find the integral.

$$\int s \ e^{3s} ds$$

$$\int \theta \cos 2\theta \ d\theta$$

$$\int x^2 \ln x \ dx$$

Repeated Integration by Parts

◆ Integration by parts may need to be ______ to evaluate an integral.

EXAMPLE

Find the indefinite integral $\int 3x^2e^x dx$.

$$\int (3x^2e^x) dx = \underline{\qquad} \cdot \underline{\qquad} - \int (\underline{\qquad} \cdot \underline{\qquad})$$

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

$$u = \underline{\hspace{1cm}} dv = \underline{\hspace{1cm}}$$

$$du = \underline{\hspace{1cm}} v = \underline{\hspace{1cm}}$$

$$u = \underline{\hspace{1cm}} dv = \underline{\hspace{1cm}}$$

$$du = \underline{\hspace{1cm}} v = \underline{\hspace{1cm}}$$

EXAMPLE

Find the indefinite integral.

(A)
$$\int (t^2 + 2t - 3) e^{4t} dt$$

$$(\mathbf{B}) \int (\ln x)^2 \, dx$$

PRACTICE

Find the indefinite integral.

$$\int r^2 e^{-r} dr$$

$$\int \theta^2 \cos 3\theta \ d\theta$$

Tabular Integration by Parts

◆ For more complicated integrals that require repeated IBP, it can be helpful to organize into a ______.

EXAMPLE

Find the integral $\int 3x^2e^{2x} dx$.

Recall	Integration by Parts	New	Tabular Integration by Parts					
	$u \cdot dv = u \cdot v - \int v \cdot du$			<i>u</i> & its D erivatives	dv & its antiderivatives (Integrals)	:		
$\int (3x^2e^{2x}) dx$ $u dv$		Alternating Signs	D	I				
	Start with:	[+ -]						
$=3x^{2}\left(\frac{1}{2}e^{2x}\right)-\int\left(\frac{1}{2}e^{2x}\cdot 6x\right)dx$ u v du		[+ -]						
		[+ -]						
		[+ -]						
$=3x^2 \cdot \frac{1}{2}e$	$\frac{1}{4}e^{2x} - \left(6x \cdot \frac{1}{4}e^{2x} - \int \frac{1}{4}e^{2x} \cdot 6 dx\right)$	$\int (3x^2e^{2x}) dx = (\underline{})(\underline{}) (\underline{})$						
u v u v v du		$(\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) \qquad \int (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) + C$						
$=\frac{3}{2}x^2e^{2x}$	$-\frac{3}{2}xe^{2x} + \frac{3}{4}e^{2x} + C$		=					

lacktriangle In the *D* column, differentiate *u* to _____ if possible or until the product of the bottom row is a basic integral.

EXAMPLE

Evaluate the indefinite integral using the tabular method.

$$\int t^3 \ln t \ dt$$

$$\int e^t \sin 3t \ dt$$

PRACTICE

Evaluate the indefinite integral.

$$(A) \int (s^2 + 4)e^{3s} \, ds$$

$$\int \frac{\ln x}{x^4} \ dx$$

$$\int x^2 \cos 4x \ dx$$

Integration by Parts: Definite Integrals

◆ To find a definite integral using IBP, solve like an indefinite integral, then apply bounds to ______ parts at the end.

EXAMPLE

Evaluate the definite integral using integration by parts.

$$\int_0^2 6xe^x dx$$

PRACTICE

Evaluate the definite integral.

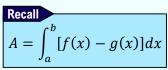
$$(A) \int_{1}^{2} x \ln x \ dx$$

Recall
$$\int_{a}^{b} u \cdot dv = u \cdot v \Big|_{a}^{b} - \int_{a}^{b} v \cdot du$$

$$\int_0^\pi t^2 \sin t \ dt$$

EXAMPLE

Find the area of the region bounded by $f(x) = (x - 1)e^{3x}$ and the *x*-axis on [1,4].



(Area Between Curves

EXAMPLE

Find the volume of the solid generated when the region bounded by $y = \sqrt{x \cos x}$, the *x*-axis, and the lines x = 0 and $x = \pi/2$ are revolved about the *x*-axis.

Recall
$$Volume = \int_{a}^{b} \pi [R(x)]^{2} dx$$

PRACTICE

The rate of growth of a particular bacteria is given by $b'(t) = 4t^2e^{.5t}$ where t is time in days. What is the total growth of the population of bacteria during the first 5 days?

Recall
$$\int_{a}^{b} u \cdot dv = u \cdot v \Big|_{a}^{b} - \int_{a}^{b} v \cdot du$$