TOPIC: RELATED RATES

Intro to Related Rates

- When 2+ related variables are changing w/ time, we use ______-derivatives to see how one affects another.
  - ► To take the time derivative of *EVERY* term, use implicit differentiation.

EXAMPLE If $y = x^3$ and $\frac{dx}{dt} = 2$, find $\frac{dy}{dt}$ when $x = 4$.

Recall Implicit Diff.	New Related Rates
$y^2 + 5x = x^3$	$y = x^3$
$\frac{d}{dx}(y^2 + 5x = x^3)$	
$2y\frac{dy}{dx} + 5 = 3x^2\frac{dx}{dx}$	
$\frac{dy}{dx} = \frac{3x^2 - 5}{2y}$	

HOW TO: Solve Related Rates

1) Take —— on both sides using implicit diff.

2) Isolate the target **rate of change**

3) Plug in known values/rates & solve

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PRACTICE Given the equation below, find $\frac{dy}{dt}$ when $\frac{dx}{dt} = 12$ and $x = \frac{15}{2}$.

$$y = \sqrt{2x + 1}$$

PRACTICE Given the equation below, find $\frac{dy}{dt}$ when $\frac{dx}{dt} = 3$, $\frac{dy}{dt} = 2$, $x = 2$, $y = 4$, and $z = 1$.

$$x^2 + y^2 - 3z^2 = 125$$

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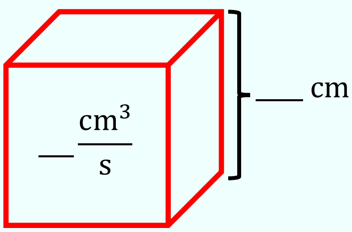
Changing Geometries

◆ Recall: Related rates problems use implicit differentiation to find specific time derivatives (e.g. $\frac{dx}{dt}, \frac{dy}{dt}, \frac{dV}{dt}$).

► Many related rates problems involve common shapes that ________ (+) or ________ (−) over time.

EXAMPLE

A cube grows at a rate of $2\,\frac{\text{cm}^3}{\text{s}}$. How fast is the side of the cube growing when it has a length of 0.8 cm?



HOW TO: Solve Related Rates

1) Draw/label a picture of the scenario

2) Identify eq'n(s) relating ___ variables

3) Take $\frac{d}{dt}$ on both sides using implicit diff.

4) Isolate the target **rate of change**

5) Plug in known values/rates & solve

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PRACTICE

A sphere is growing at a rate of $50\frac{cm^3}{s}$. At what rate is the radius of the sphere increasing when the radius is $5\ cm$?

PRACTICE

A right triangle has a base of $10\ cm$ and a height of $12\ cm$. The height of the right triangle is decreasing at a rate of $0.4\frac{cm}{s}$, at what rate is the area of the triangle decreasing?

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PRACTICE

The perimeter of a rectangle is fixed at $30\ cm$. If the length L is increasing at a rate of $2\,\frac{cm}{s}$, for what value of L does the area start to decrease? Hint: The rectangle's area starts to decrease when the rate of change for the area is less than 0.

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Real World Applications

◆ Recall: Most related rates problems have shapes that grow $(+)$ or shrink $(-)$.

► For "real world" problems, we need to determine *what* shapes are formed & *how* they change over time.

EXAMPLE

As an ice cube melts, each side changes at $-3\,\frac{\text{cm}}{\text{min}}$. Find the rate of change of the ice cube's volume when each side is 0.9 cm.

HOW TO: Solve Related Rates

1) Draw/label a picture of the scenario

2) Identify eq'n(s) relating **ALL** variables

3) Take $\frac{d}{dt}$ on both sides using implicit diff.

4) Isolate the target **rate of change**

5) Plug in known values/rates & solve

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PRACTICE

A 15-foot plank leans against a vertical pole. The top of the plank begins to slide down the pole at a steady speed of 2 inches per second. How fast is the bottom of the plank moving away from the pole when it is 8 feet away from the base of the pole (in inches per second)?

PRACTICE

Two cars leave the same intersection and drive in perpendicular directions. Car A travels east at a speed of $60\frac{mi}{hr}$, and Car B travels north at a speed of $40\frac{mi}{hr}$. Car A leaves the intersection at 2pm, while Car B leaves at 2:30pm. Determine the rate at which the distance between the two cars is changing at 3pm.