

TOPIC: THE CHAIN RULE

Intro to the Chain Rule

◆ To differentiate a composite function $f(g(x))$, start from the **outside** and work your way **inside**.

| RULES OF DIFFERENTIATION | | |
|--------------------------|---|--|
| Name | Rule | Example |
| Chain | $\frac{d}{dx}[f(g(x))] = \text{---}(\text{---}) \cdot \text{---}$ | $\frac{d}{dx}(4x + 5)^3 = \text{---}(\text{---}) \cdot \text{---}$ |

► Alternate notation: If $y = f(u)$ and $u = g(x)$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

EXAMPLE

Find $f'(x)$ using the chain rule.

$$f(x) = 2(3x^2 - x)^4$$

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PRACTICE

Find the derivative of the function.

(A)

$$f(x) = \sqrt{5x^2 - 3x}$$

(B)

$$y = (8x^3 - 2x)^{3/2}$$

(C)

$$f(t) = (3t^2 + 7t - 2)^{10}$$

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EXAMPLE

Find $\frac{dy}{dx}$ for the function.

(A)

$$y = (2x - 1)^4 \cdot (3 + x)^2$$

(B)

$$y = \frac{(2x - 1)^4}{(3 + x)^2}$$

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EXAMPLE

Find the derivative of each function.

(A) $f(x) = \sin^5 x$

(B) $f(x) = \sin(x^5)$

PRACTICE

Find the derivative of the function.

(A) $f(x) = \sin(3x^2)$

(B) $y = 3\cos^4 \theta$

(C) $f(t) = \sec(4t + 5)$

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The Chain Rule for 3+ Functions

◆ Recall: Use the chain rule from the **outside** to **inside**. You may need to use this rule *multiple* times.

► When given trig functions with powers (e.g. $\sin^n x$), rewrite power on the _____: $(\sin x)^{\text{—}}$

EXAMPLE

Find $f'(x)$ using the chain rule.

$$f(x) = \sin^4(3x^2)$$

Recall

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

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PRACTICE

Find the derivative of the function.

(A)

$$f(x) = \sin^5(2x^3 + 1)$$

(B)

$$y = \cos^3(\sec \theta)$$

EXAMPLE

Find the derivative.

$$f(x) = \tan(3 - \sin 2x)$$