TOPIC: THE SECOND DERIVATIVE TEST

The Second Derivative Test: Finding Local Extrema

◆ Recall: The sign of the second derivative f'' tells us whether a function is concave up $(+)$ or down $(-)$.

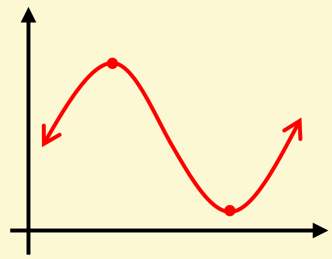
► Determine whether a ___________ point is a local max or min based on its **concavity** at that point.

Second Derivative Test

Suppose f'' is continuous on an open interval containing c.

If $f'(c) =$ ____ AND…

- $f''(c)$ is ___, then f has a local [**MIN | MAX**] at c.
- $f''(c)$ is ___, then f has a local [**MIN | MAX**] at c.
- $f''(c)$ is ___, then ___, use ________ derivative test.



EXAMPLE

Locate the local extrema of $f(x)$ using the second derivative test.

$f(x) = x^3 - 3x^2 + 4$

HOW TO: Find Local Extrema Using Second Derivative Test

1) Find where $f'(x) =$ ____*

2) Plug values from **(1)** into f''. If…

f'' is*: $-$, pt. is local **MAX**

$+$, pt. is local **MIN**

3) If asked: Find **value** of max/min by plugging crit. pt. into _____

If $f'(x)$ DNE or $f'' = 0$, use 1st deriv. test

TOPIC: THE SECOND DERIVATIVE TEST

PRACTICE Use the second derivative test to find the local extrema of the given function.

HOW TO: Find Local Extrema Using Second Derivative Test

1) Find where $f'(x) = \mathbf{0}$*

2) Plug values from **(1)** into f''. If…

$\boldsymbol{f''}$ is*: $-$, pt. is local **MAX**

$+$, pt. is local **MIN**

3) If asked: Find **value** of max/min by plugging crit. pt. into $\boldsymbol{f(x)}$

**If $f'(x)$ DNE or $f'' = 0$, use 1st deriv. test*

$(\boldsymbol{A})$

$g(x) = x^3 - 6x^2 + 9x + 2$

$(\boldsymbol{B})$

$$f(x) = \frac{x^2 - 4}{x^2 + 1}$$

$(\boldsymbol{C})$

$f(x) = 4 \sin x \cos x;\ 0 < x < \pi$

TOPIC: THE SECOND DERIVATIVE TEST

EXAMPLE Find the local extrema of $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$.

HOW TO: Find Local Extrema Using Second Derivative Test

1) Find where $f'(x) = \mathbf{0}$*

2) Plug values from **(1)** into f''. If…

$\boldsymbol{f''}$ is*: $-$, pt. is local **MAX**

$+$, pt. is local **MIN**

3) If asked: Find **value** of max/min by plugging crit. pt. into $\boldsymbol{f(x)}$

If $f'(x)$ DNE or $f'' = 0$, use 1st deriv. test