TOPIC: INTEGRALS INVOLVING LOGARITHMIC FUNCTIONS

Integrals Resulting in Natural Logs

◆ The *derivative* of $\ln x$ is **Recall** $\frac{d}{dx}\ln x = \frac{1}{x}, x > 0$ & we can _____ this rule to find the *integral* of $\frac{1}{x}$.

New **The Integral Resulting in Natural Log**

$$\int \frac{1}{x}\,dx = \_\_\_\_\_ + C$$

$$\int \frac{5}{x}\,dx =$$

◆ Remember, we can use the power rule to find the integral of x^{-n} when $n \neq 1$.

EXAMPLE Find the indefinite integral.

$$\int \left(\frac{1}{x^2} + \frac{3}{x}\right) dx$$

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EXAMPLE Find the indefinite integral.

$$\int \frac{x^3 e^x - 4x^2}{x^3}\, dx$$

Recall

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

PRACTICE Find the indefinite integral.

$$\int \frac{3 - y^2}{2y}\, dy$$

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EXAMPLE Evaluate the definite integral.

$$\int_1^5 \frac{dt}{4t}$$

Recall

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

PRACTICE Evaluate the definite integral.

$$\int_2^e \frac{3+x}{x}\,dx$$

PRACTICE Find the area under the graph of $f(t) = \frac{t^2 e^t + t}{t^2}$ from $t = 1$ to $t = 3$.

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Integrals Involving Natural Logs: Substitution

◆ Recall: The integral of $\frac{1}{x}$ is

Recall

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

► To evaluate the integrals of more complicated rational functions, first try **substitution** where $u =$ _____.

EXAMPLE Find the indefinite integral $\int \frac{4}{3+4x}\,dx$.

New **Integrals Resulting in Natural Log: Substitution**

$$\int \frac{4}{3+4x}\,dx =$$

$$\int \frac{1}{u}\,du = \_\_\_\_\_ + C$$

EXAMPLE Evaluate the given integrals.

$(\boldsymbol{A})$

$$\int \frac{2x+4}{x^2+4x}\,dx$$

$(\boldsymbol{B})$

$$\int \frac{5\,dx}{x\,(\ln x)^3}$$

◆ If $u =$ denominator does not work, you may need to try a different substitution or polynomial division.

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PRACTICE Find the indefinite integral.

$$\int \frac{1}{2x+5}\,dx$$

Recall

$$\int \frac{1}{u}\,du = \ln|u| + C$$

EXAMPLE Find the antiderivative.

$$\int \frac{e^{2x}}{1+e^{2x}}dx$$

EXAMPLE Find the antiderivative.

$$\int \frac{2dx}{x\ln x}$$

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EXAMPLE Find the following antiderivatives.

(***A***)

$$\int \tan x \; dx$$

(***B***)

$$\int \cot x \; dx$$

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EXAMPLE Find the following antiderivatives.

(***A***)

$$\int \sec x \; dx$$

Hint: Multiply by $\frac{\sec x+\tan x}{\sec x+\tan x}$

(***B***)

$$\int \csc x \; dx$$

Hint: Multiply by $\frac{\csc x+\cot x}{\csc x+\cot x}$

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EXAMPLE Find the indefinite integral.

$$\int \frac{x^2 + 2x + 3}{x + 3}\, dx$$

EXAMPLE Compute the indefinite integral.

$$\int \frac{x}{(x - 2)^2}\, dx$$

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PRACTICE Evaluate the definite integral.

$(\boldsymbol{A})$

$$\int_1^6 \frac{dt}{2+5t}$$

$(\boldsymbol{B})$

$$\int_0^{\pi/3} \frac{\sin\theta}{1+\cos\theta}\, d\theta$$

PRACTICE Find the area under the graph of $f(x) = \frac{e^{-x}}{1+e^{-x}}$ from $x = 0$ to $x = 4$.

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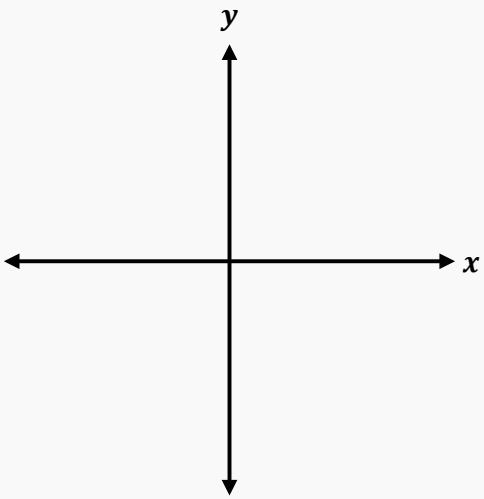
PRACTICE

The region between the curve $y = \frac{2}{\sqrt{x}}$ and the x-axis from $x = 1$ to $x = 3$ is revolved about the x-axis to form a solid. Find the volume of this solid.

Recall

$$Volume = \int_a^b \pi[R(x)]^2 dx$$



PRACTICE

Solve the initial value problem given by $\frac{dy}{dx} = \frac{2}{x} + 3, y(1) = 4.$