TOPIC: APPLIED OPTIMIZATION

Intro to Applied Optimization: Maximizing Area

- Recall: Find **maximum/minimum** values of a function by testing critical & end points **OR** using the 2nd derivative.
  - **Optimize** a "real world" value by expressing it as a ____ based on given *constraints*, then finding the **max/min**.

EXAMPLE

Given 200 ft of fencing to construct a rectangular fence, determine the dimensions that would create the maximum area where one side is formed by a rock wall & does not need fencing.

HOW TO: Solve Optimization Problems

1) Draw diagram & identify ___________

2) a. Write fcn val for being ___________

b. Put in terms of ___ variable*

3) Determine _________ restrictions

4) Find critical pts ($f'(x) =$ ___)

5) a. If endpoints (______ interval):

Plug critical & end pts into **orig fcn**

Largest value = ______

Smallest value = ______

b. If **NO** endpoints (____ interval):

Plug critical pt into **2nd derivative**

Negative $(-)$: ______

Positive $(+)$: ______

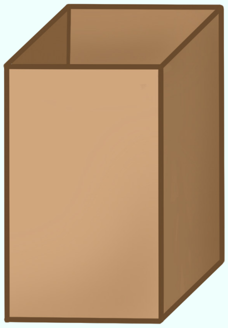
**May require additional eqns*

TOPIC: APPLIED OPTIMIZATION

Minimizing Surface Area

EXAMPLE

A shipping company wants to construct a rectangular box with a square base to have a volume of 125 in^3. If the box has no lid on, what should the dimensions be to use the least amount of cardboard?



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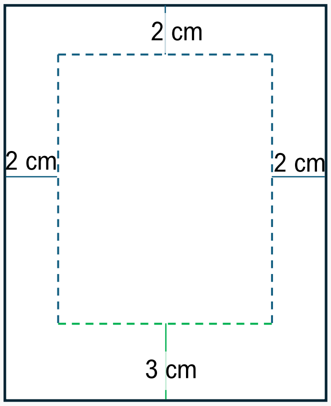
PRACTICE

A poster is set to have a total area of 1150 cm^2, with 2-cm margins on the sides and the top, and a 3-cm margin at the bottom. What dimensions will maximize the printed area?



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TOPIC: APPLIED OPTIMIZATION

Maximizing Revenue & Profit

◆ To maximize revenue $R(x)$ or profit $P(x)$, put given information into known equations & find maximum.

EXAMPLE

A coffee shop sells x coffees a day at $\$p$ per coffee. Given the price-demand function $p(x) = 100 - \frac{x}{4}$, how many coffees does the shop need to sell to maximize their daily revenue?
Setup, but do not solve.

$R(x)$	$=$	$p(x)$	$\times$	x
Revenue		*Price*		*Items Sold*

$P(x)$	$=$	$R(x)$	$-$	$C(x)$
Profit		*Revenue*		*Cost*

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Maximizing Profit

EXAMPLE

Gerald and Katya recently started selling custom-made phone cases. They've noticed that the **price per case** they can charge depends on how many cases they sell, which is represented by the following relationship.

$$p(x) = 50 - 0.2x$$

where x is the no. of cases sold, and $p(x)$ is the price per case in dollars. On top of that, the **cost** of making x cases is given by:

$$C(x) = 100 + 15x$$

They want to maximize their profit. In such case, find:

(a) the number of cases that should be sold to result in maximum profit

(b) the maximum profit

(c) the corresponding price

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TOPIC: APPLIED OPTIMIZATION

PRACTICE

Your café sells lattes for $4 each to 100 customers per day. For every $1 increase in price, you would lose 20 customers. Find the price that maximizes revenue.

Hint: The # of items sold is based on the number of customers.

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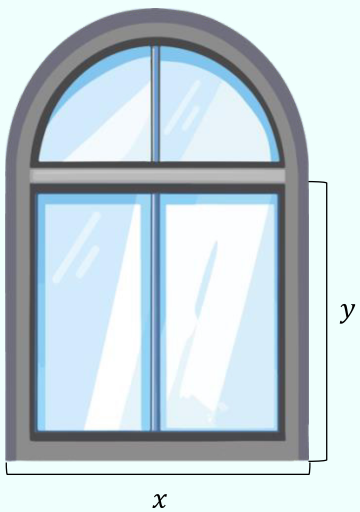
Norman Window

EXAMPLE

A Norman window is shaped as a rectangle with a semicircle on top. The perimeter of the window is 24 meters, and the semicircular part lets in one quarter as much light per square meter as the rectangular part. What dimensions of the window (width and height of the rectangle) will allow the maximum amount of light to pass through? Consider the window trim negligible.



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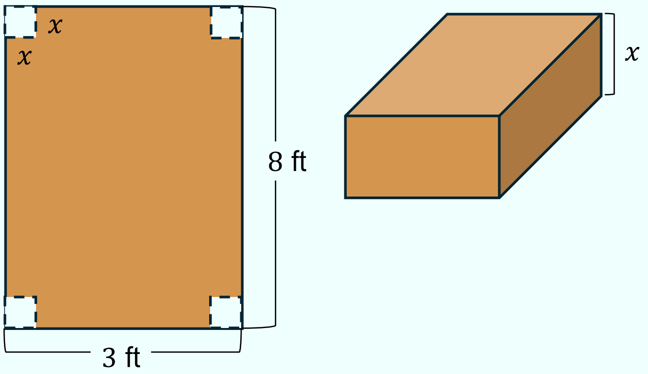
Packaging Design

EXAMPLE

A box with an open top is formed by cutting equal-sized squares from the corners of a 3 ft by 8 ft rectangular sheet of cardboard and folding up the sides. What dimensions of the cut squares would give the box the maximum possible volume, and what is that volume?



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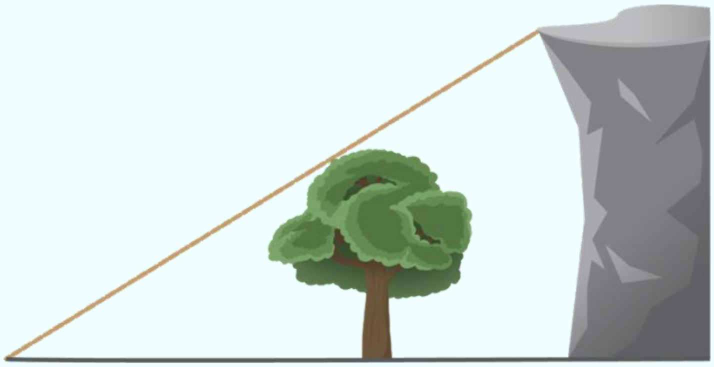
Shortest Rope

EXAMPLE

A 10-meter-tall tree stands in front of a rock cliff, with its base 5 meters away from the base of the cliff. What is the length of the shortest rope that can be tied from the ground over the top of the tree to the top of the cliff?



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