

## TOPIC: INTRODUCTION TO LOGARITHMS

### Logarithms Introduction

- ◆ The \_\_\_\_\_ (inverse) operation of an exponential is taking the **logarithm** (log).
  - Logs and exponentials with the same **base** \_\_\_\_\_ each other.
  - A **log** gives us the **power** that some **base** must be raised to in order to equal a particular number.

#### Solving Polynomials

$$\begin{aligned}
 x^3 &= 216 \\
 \sqrt[3]{x^3} &= \sqrt[3]{216} \\
 x &= \sqrt[3]{216}
 \end{aligned}$$

(A)

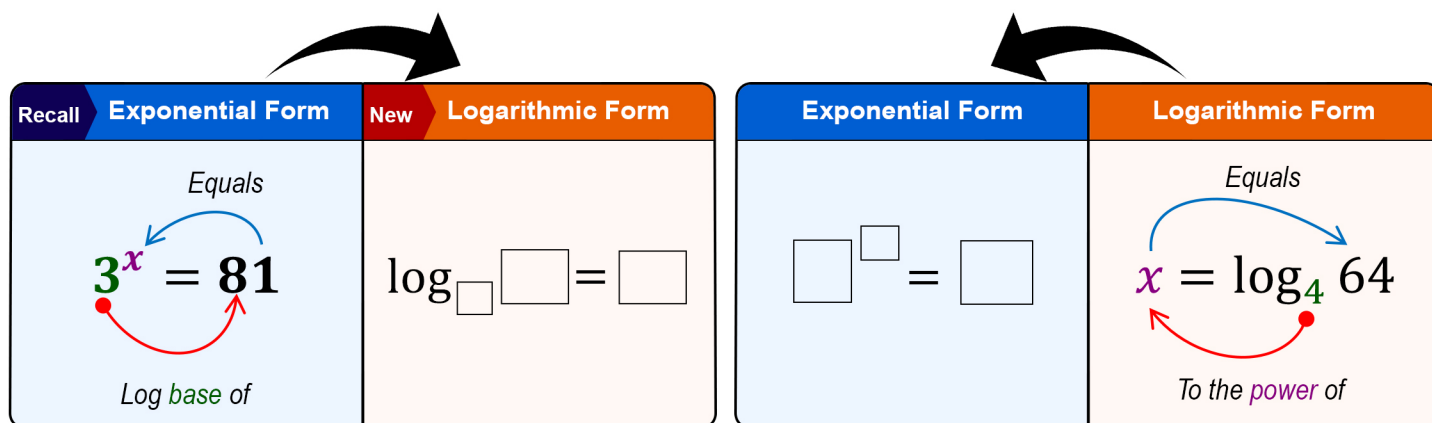
$$\begin{aligned}
 2^x &= 8 \\
 2 \times 2 \times 2 &= 8 \\
 x &= 3
 \end{aligned}$$

#### Solving Exponentials

(B)

$$\begin{aligned}
 2^x &= 216 && \text{(Exponential Form)} \\
 x &= \log && \text{(Logarithmic Form)} \\
 &&& \text{"log base 2 of 216"}
 \end{aligned}$$

- ◆ You will need to convert expressions between **exponential form** and **logarithmic form**.



#### EXAMPLE

Write each log in exponential form & each exponential in log form.

(A)

$$x = \log_5 800$$

(B)

$$\log_2 16 = 4$$

(C)

$$10^x = 4500$$

- ◆  $\log_{10}$ , known as the \_\_\_\_\_ log, can be written as just \_\_\_\_\_ and has its own calculator button: **LOG**

## TOPIC: GRAPHING LOGARITHMIC FUNCTIONS

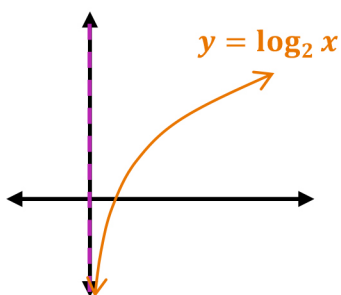
### Graphs of Logarithmic Functions

- ◆ We can graph a logarithmic function using the fact that it is the \_\_\_\_\_ of an exponential function.
- $f(x) = \log_b x$  can be graphed by \_\_\_\_\_ the graph of its inverse function,  $y = b^x$  over \_\_\_\_\_.

Recall	Exponential Functions	New	Logarithmic Functions																												
	<table border="1"> <thead> <tr> <th><math>x</math></th> <th><math>f(x) = 2^x</math></th> </tr> </thead> <tbody> <tr><td>-2</td><td><math>\frac{1}{4}</math></td></tr> <tr><td>-1</td><td><math>\frac{1}{2}</math></td></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>4</td></tr> <tr><td>3</td><td>8</td></tr> </tbody> </table>	$x$	$f(x) = 2^x$	-2	$\frac{1}{4}$	-1	$\frac{1}{2}$	0	1	1	2	2	4	3	8		<table border="1"> <thead> <tr> <th><math>x</math></th> <th><math>f(x) = \log_2 x</math></th> </tr> </thead> <tbody> <tr><td><math>\frac{1}{4}</math></td><td></td></tr> <tr><td><math>\frac{1}{2}</math></td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td>2</td><td></td></tr> <tr><td>4</td><td></td></tr> <tr><td>8</td><td></td></tr> </tbody> </table>	$x$	$f(x) = \log_2 x$	$\frac{1}{4}$		$\frac{1}{2}$		1		2		4		8	
$x$	$f(x) = 2^x$																														
-2	$\frac{1}{4}$																														
-1	$\frac{1}{2}$																														
0	1																														
1	2																														
2	4																														
3	8																														
$x$	$f(x) = \log_2 x$																														
$\frac{1}{4}$																															
$\frac{1}{2}$																															
1																															
2																															
4																															
8																															
	Domain: <i>always</i> $\mathbb{R}$ Range: depends on asymp.; $(0, \infty)$ Horizontal <b>Asymptote</b> at $y = 0$		Domain: depends on _____; _____ Range: <i>always</i> _____ _____ <b>Asymptote</b> at _____																												

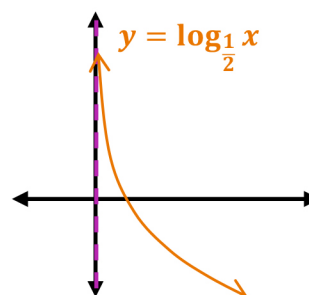
- ◆ Just like its inverse, the direction of the graph of  $f(x) = \log_b x$  depends on \_\_\_\_.

$$b > 1$$



- ◆ Graph [ INCREASES | DECREASES ]

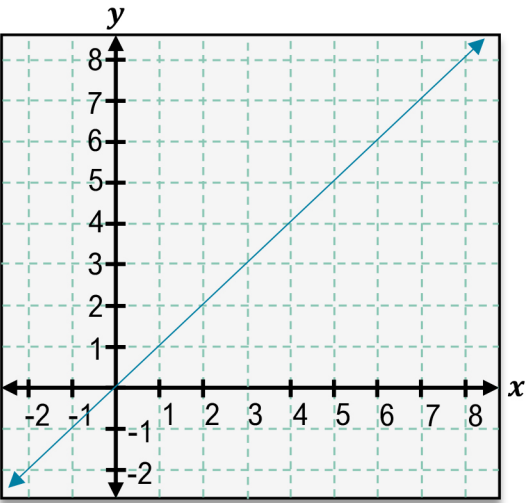
$$0 < b < 1$$



- ◆ Graph [ INCREASES | DECREASES ]

**TOPIC: GRAPHING LOGARITHMIC FUNCTIONS**

**EXAMPLE:** Graph  $f(x) = 3^x$  and  $g(x) = \log_3 x$  on the graph below. Determine the domain and range of each.



$x$	$f(x) = 3^x$
-2	
-1	
0	
1	
2	

Domain: \_\_\_\_\_  
Range: \_\_\_\_\_


$x$	$g(x) = \log_3 x$

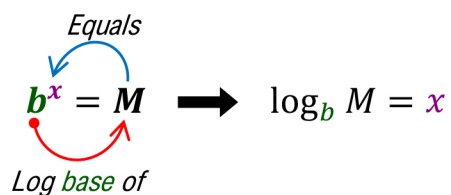
Domain: \_\_\_\_\_  
Range: \_\_\_\_\_

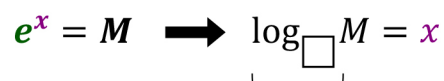
## TOPIC: INTRODUCTION TO LOGARITHMS

### The Natural Log

◆ Besides the common log, ( $\log_{10}$ ), another frequently occurring log is  $\log$ \_\_\_\_, called the \_\_\_\_\_ log.

- The natural log is written as \_\_\_\_\_, and also has its own calculator button: 


$$b^x = M \longrightarrow \log_b M = x$$


$$e^x = M \longrightarrow \log_{\boxed{e}} M = x$$

### EXAMPLE

Write each log in exponential form & each exponential in log form.

(A)  $x = \ln 17$

(B)  $e^x = 4$

## TOPIC: INTRODUCTION TO LOGARITHMS

### PRACTICE

Convert the following logarithmic statement to its equivalent exponential form.

(A)

$$\log_4 x = 5$$

(B)

$$x = \log 9$$

### PRACTICE

Convert the following exponential statements to their equivalent logarithmic form.

(A)

$$3^x = 7$$

(B)

$$e^9 = x + 3$$