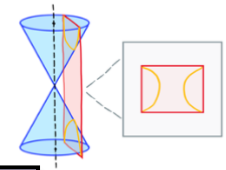
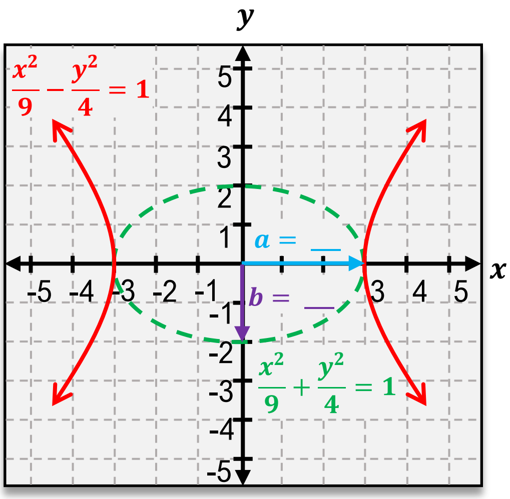
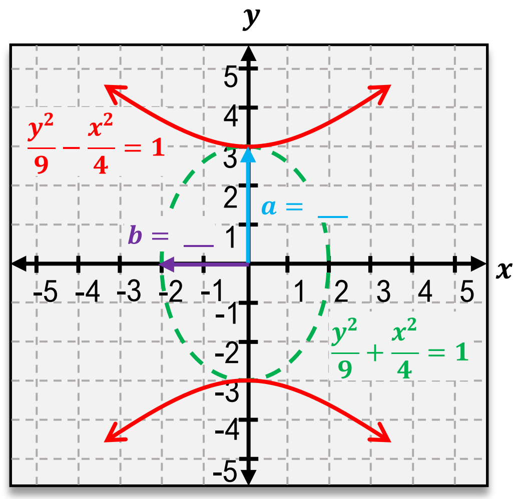
TOPIC: HYPERBOLAS AT THE ORIGIN

Introduction to Hyperbolas

Circle	Ellipse	Parabola	**Hyperbola**



- The equation for a hyperbola is the same as an ellipse, but with a ______.
  - Visually, a hyperbola appears as two ____________ facing away from each other.

Horizontal Hyperbola	Vertical Hyperbola
	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (*Horiz. Ellipse*) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (*Horiz. Hyperbola*)	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ (*Vert. Ellipse*) $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ (*Vert. Hyperbola*)

- The axis major axis (a) is always **[LARGEST | FIRST]** for an ellipse, and **[LARGEST | FIRST]** for a hyperbola.

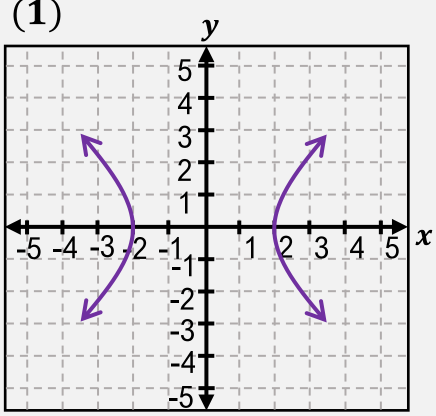
EXAMPLE: Match the equation to the graph

(A) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

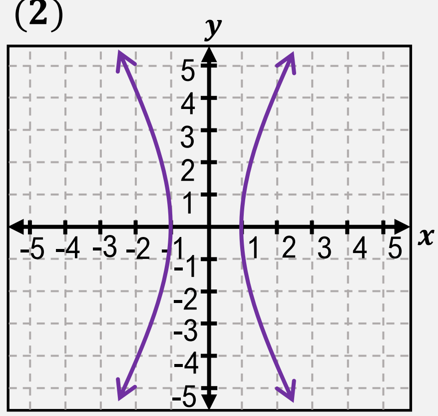
(B) $\frac{x^2}{4} - \frac{y^2}{1} = 1$

(C) $\frac{x^2}{1} - \frac{y^2}{25} = 1$

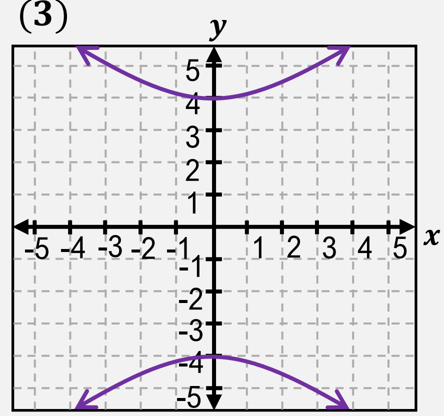
(1)



(2)



(3)



TOPIC: HYPERBOLAS AT THE ORIGIN

PRACTICE: Given the hyperbola $\frac{x^2}{25}-\frac{y^2}{9}=1$, find the length of the a-axis and b-axis.

$(\boldsymbol{A})\ a=25, b=9$
$(\boldsymbol{B})\ a=9, b=25$
$(\boldsymbol{C})\ a=5, b=3$
$(\boldsymbol{D})\ a=3, b=5$

PRACTICE: Given the hyperbola $x^2-\frac{y^2}{4}=1$, find the length of the a-axis and b-axis.

$(\boldsymbol{A})\ a=1, b=4$
$(\boldsymbol{B})\ a=4, b=1$
$(\boldsymbol{C})\ a=1, b=2$
$(\boldsymbol{D})\ a=2, b=1$

Circle	Ellipse	Parabola	Hyperbola

TOPIC: HYPERBOLAS AT THE ORIGIN

PRACTICE: Given the hyperbola $\frac{y^2}{100} - \frac{x^2}{139} = 1$, find the length of the a-axis and b-axis.

$(\boldsymbol{A})\ a = 100, b = 139$
$(\boldsymbol{B})\ a = 139, b = 100$
$(\boldsymbol{C})\ a = \sqrt{139}, b = 10$
$(\boldsymbol{D})\ a = 10, b = \sqrt{139}$

TOPIC: HYPERBOLAS AT THE ORIGIN

Vertices and Foci of Hyperbolas

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(*Horiz. Hyperbola*)

- Every hyperbola has 2 **Vertices** & 2 **Foci**, both located on the **[MAJOR | MINOR]** axis.
  - **Vertices** are the points on the hyperbola __________ to the center

    Distance between center & **Vertex** = **[a | b | c]**
  - For any point on a hyperbola, the __________ of the distances between the point & each **Focus** is a constant

    Distance between center & **Focus** = **[a | b | c]**

$$c^2 = a^2 - b^2$$

(*Foci of Ellipse*)

$$c^2 = a^2 \quad\quad b^2$$

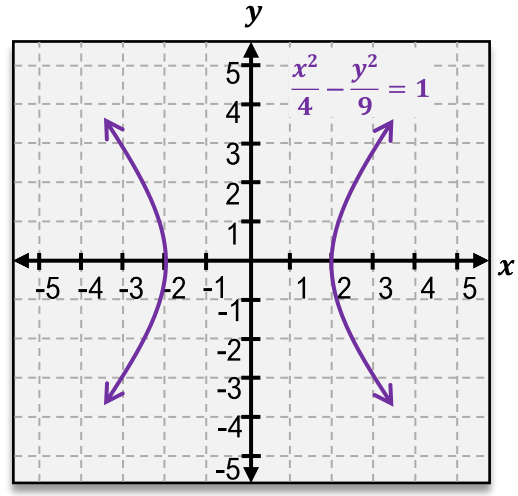
(*Foci of Hyperbola*)

EXAMPLE: Find the vertices & foci of the hyperbola in the graph.

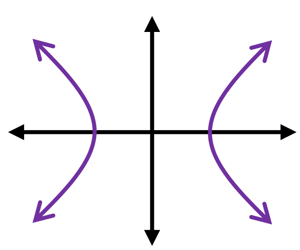
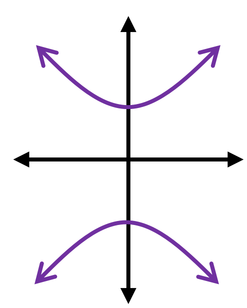
$a =$

$b =$

$c =$



- For a *vertical* hyperbola, the coordinates of vertices & foci are different

Horizontal Hyperbola	Vertical Hyperbola
	
Vertices: (___ ,0) & (___ , 0)	Vertices: (0 , ___) & (0 , ___)
Foci: (___ , 0) & (___ , 0)	Foci: (0, ___) & (0, ___)
Vertices & Foci on **[x | y]** axis	Vertices & Foci on **[x | y]** axis

Circle	Ellipse	Parabola	**Hyperbola**

TOPIC: HYPERBOLAS AT THE ORIGIN

PRACTICE: Determine the vertices and foci of the hyperbola $\frac{y^2}{4} - x^2 = 1$.

$(\boldsymbol{A})$ Vertices: $(2,0), (-2,0)$
Foci: $(\sqrt{5}, 0), (-\sqrt{5}, 0)$
$(\boldsymbol{B})$ Vertices: $(0,2), (0,-2)$
Foci: $(0, \sqrt{5}), (0, -\sqrt{5})$
$(\boldsymbol{C})$ Vertices: $(1,0), (-1,0)$
Foci: $(5,0), (-5,0)$
$(\boldsymbol{D})$ Vertices: $(0,1), (0,-1)$
Foci: $(0,5), (0,-5)$

PRACTICE: Find the equation for a hyperbola with a center at $(0,0)$, focus at $(0,-6)$ and vertex at $(0,4)$.

$(\boldsymbol{A})$ $\frac{y^2}{16} - \frac{x^2}{20} = 1$
$(\boldsymbol{B})$ $\frac{y^2}{20} - \frac{x^2}{16} = 1$
$(\boldsymbol{C})$ $\frac{y^2}{4} - \frac{x^2}{\sqrt{20}} = 1$
$(\boldsymbol{D})$ $\frac{y^2}{\sqrt{20}} - \frac{x^2}{4} = 1$

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Asymptotes of Hyperbolas

Circle	Ellipse	Parabola	Hyperbola

- To graph hyperbolas, you'll need asymptotes
  - The values of a & b form a ______ where the asymptotes are draw through the corners of the shape.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

(Vert. Hyperbola)

$$y = \pm\frac{}{}x$$

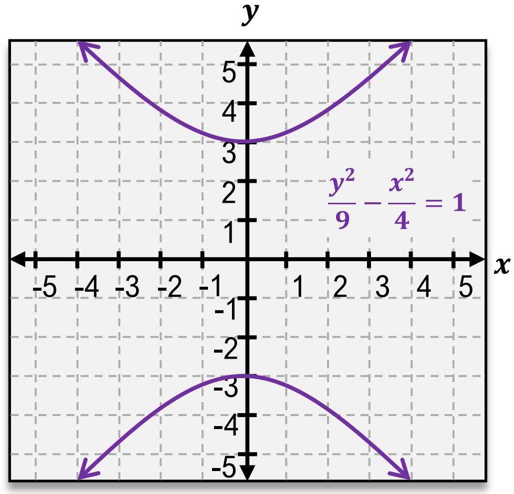
(Vert. Hyperbola Asym.)

EXAMPLE: Find & draw the asymptotes of the given hyperbola.

$a =$

$b =$

Asymptotes: $y =$ ________ & $y =$ ________



- For asymptotes of horizontal hyperbolas, just flip a & b

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(Horiz. Hyperbola)

$$y = \pm\frac{}{}x$$

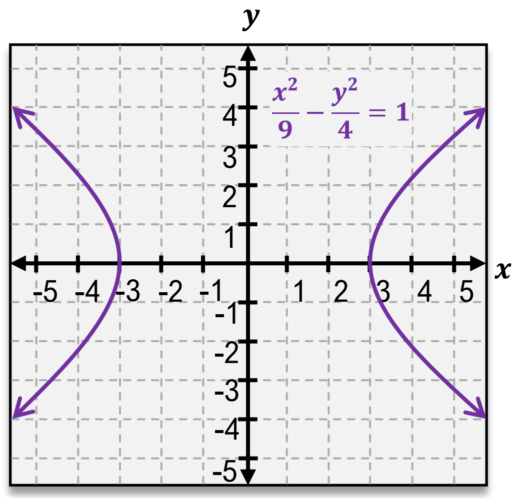
(Horiz. Hyperbola Asym.)

EXAMPLE: Find & draw the asymptotes of the given hyperbola.

$a =$

$b =$

Asymptotes: $y =$ ________ & $y =$ ________



PRACTICE: Find the equations for the asymptotes of the hyperbola $\frac{x^2}{64} - \frac{y^2}{100} = 1$.

$(\boldsymbol{A})\ y = \pm\frac{4}{5}x$

$(\boldsymbol{B})\ y = \pm\frac{5}{4}x$

$(\boldsymbol{C})\ y = \pm\frac{16}{25}x$

$(\boldsymbol{D})\ y = \pm\frac{25}{16}x$

Circle	Ellipse	Parabola	Hyperbola

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PRACTICE: Find the equations for the asymptotes of the hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$.

$(\boldsymbol{A})\ y = \pm\frac{9}{16}x$

$(\boldsymbol{B})\ y = \pm\frac{16}{9}x$

$(\boldsymbol{C})\ y = \pm\frac{3}{4}x$

$(\boldsymbol{D})\ y = \pm\frac{4}{3}x$

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Circle	Ellipse	Parabola	**Hyperbola**

EXAMPLE**:** Graph the hyperbola and identify the foci.

$$\frac{x^2}{9} - \frac{y^2}{64} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

TO GRAPH

1) Hyperbola is **[** **HORIZONTAL** **|** **VERTICAL** **]**

2) **Vertices** $(\pm a, 0)$, *OR* $(0, \pm a)$**:**

(__ , __) & (__ , __)

3) ***b* points** $(\pm b, 0)$, *OR* $(0, \pm b)$**:**

(__ , __) & (__ , __)

4) Asymptotes:

(A) draw a box through **vertices** & ***b* points**

(B) draw lines through box corners

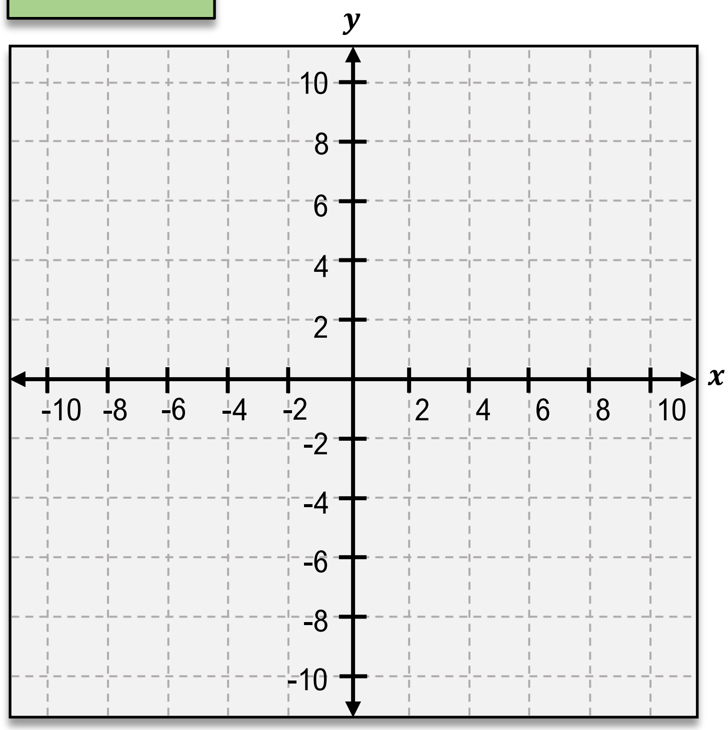
5) Draw branches at **vertices** & approaching asym.

FROM GRAPH

6) Foci $(\pm c, 0)$, *OR* $(0, \pm c)$**:**

$$c^2 = a^2 + b^2$$

(__ , __) & (__ , __)



Circle | Ellipse | Parabola | **Hyperbola**

TOPIC: HYPERBOLAS AT THE ORIGIN

EXAMPLE: Graph the hyperbola and identify the foci.

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

TO GRAPH

1) Hyperbola is **[** HORIZONTAL | VERTICAL **]**

2) **Vertices** $(\pm a, 0)$, *OR* $(0, \pm a)$:

(__ , __) & (__ , __)

3) ***b* points** $(\pm b, 0)$, *OR* $(0, \pm b)$:

(__ , __) & (__ , __)

4) Asymptotes:

(A) draw a box through **vertices** & ***b* points**

(B) draw lines through box corners

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FROM GRAPH

6) Foci $(\pm c, 0)$, *OR* $(0, \pm c)$:

(__ , __) & (__ , __)