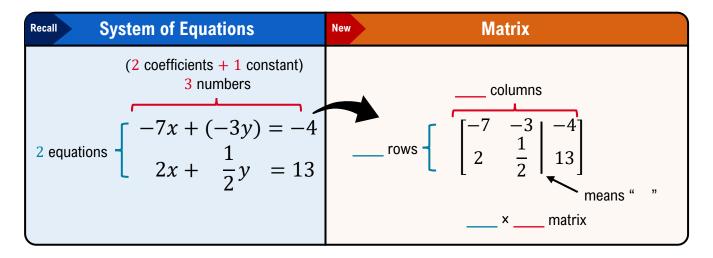
Introduction to Matrices

- ◆ A **Matrix** is a way to _____ numbers into rows & columns.
 - ▶ An Augmented Matrix represents a system of linear equations.



EXAMPLE

Represent the system of equations using an augmented matrix.

$$2x + (-3y) + z = -4$$
$$6x + 3y = 13$$
$$y - z = 8$$

PRACTICE Write the equations in standard form, then represent the system using an augmented matrix.

$$3x + 5y - 9 = 0$$
$$8x = -4y + 3$$

PRACTICE Write the system of equations represented by the augmented matrix shown.

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 5 & 4 & 1 \\ 4 & 7 & 0 & 12 \end{bmatrix}$$

Performing Row Operations on Matrices

◆ Just as we did *operations* to eqn's when solving **Systems of Eqn's**, we'll do *operations* on the _____ of a matrix.

| New | Matrix Row Operations | | | |
|--|------------------------------|---|---|--|
| Row Operation | System of Equations | Matrix | Notation | |
| CIMAD to a source | -x + 2y = 9 $2x + 6y = 12$ | $\begin{bmatrix} -1 & 2 & 9 \\ 2 & 6 & 12 \end{bmatrix}$ | Old row New row | |
| SWAP two rows | 2x + 6y = 12 $-x + 2y = 9$ | $\begin{bmatrix} 2 & 6 & 12 \\ -1 & 2 & 9 \end{bmatrix}$ | $r_1 \leftrightarrow R_2$ | |
| MIII TIDI V | 2x + 6y = 12 $-x + 2y = 9$ | $\begin{bmatrix} 2 & 6 & & 12 \\ -1 & 2 & & 9 \end{bmatrix}$ | | |
| MULTIPLY one row by any nonzero # | 2x + 6y = 12 $-2x + 4y = 18$ | | $2 \cdot r_2 \to R_2$ (Only affects row 2) | |
| | 2x + 6y = 12 $-2x + 4y = 18$ | $\begin{bmatrix} 2 & 6 & & 12 \\ -2 & 4 & & 18 \end{bmatrix}$ | 1.1 P | |
| ADD a nonzero multiple of one row to another | 10y = 30 | $\begin{bmatrix} 2 & 6 & 12 \\ 0 & 10 & 30 \end{bmatrix}$ | $\frac{r_2 + 1 \cdot r_1 \to R_2}{\text{(Only affects row 2)}}$ | |

EXAMPLE

Perform the given row operation on the augmented matrix and write the new matrix.

$$\begin{bmatrix} 2 & -6 & 4 & 10 \\ 3 & 8 & -7 & 0 \\ -1 & 5 & 9 & 3 \end{bmatrix}$$

(A)
$$r_2 \leftrightarrow R_3 \qquad \frac{1}{2} \cdot r_1 \to R_1 \qquad (C)$$

$$r_2 + 3 \cdot r_3 \to R_2$$

PRACTICE Perform the indicated Row Operation.

$$\begin{bmatrix} 7 & 9 & 3 & 4 \\ 1 & 8 & 4 & 3 \\ 2 & 0 & 5 & 2 \end{bmatrix}$$

SWAP
$$R_1 \leftrightarrow R_2$$

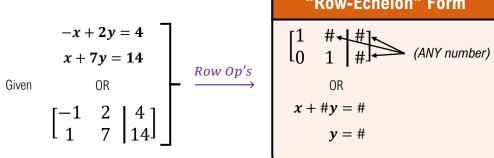
PRACTICE Perform the indicated **Row Operation**.

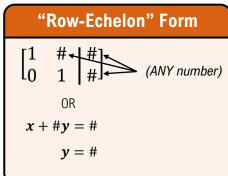
$$\begin{bmatrix} 2 & 5 & 4 & 3 \\ 5 & 3 & 1 & 2 \\ 1 & 2 & 3 & 3 \end{bmatrix}$$

$$\mathbf{ADD}\ R_1 + 2 \cdot R_3 \to R_1$$

Solving a System of Equations Using Matrices (Row - Echelon Form)

- ◆ You may have to solve a system of equations expressed as a matrix.
 - ► Use Row Operations to obtain a matrix with 1's ______ the diagonal & 0's _____ the diagonal.





- ◆ Tips for Solving:
 - 1) Work row-by-row from top-to-bottom
 - 2) Whenever you get a 1 along the diagonal, get everything under it to be 0 BEFORE trying to get the next 1

EXAMPLE

Solve the system of equations using matrices and row operations.

$$-x + 2y = 4$$
$$x + 7y = 14$$

| Matrix Row Operations | | |
|--|------------------------------|--|
| Row Operation | Sys of Eqn's | Matrix |
| SWAP two rows | -x + 2y = 9 $2x + 6y = 12$ | $\begin{bmatrix} -1 & 2 & 9 \\ 2 & 6 & 12 \end{bmatrix}$ |
| | 2x + 6y = 12 $-x + 2y = 9$ | $\begin{bmatrix} 2 & 6 & 12 \\ -1 & 2 & 9 \end{bmatrix}$ |
| MULTIPLY row by nonzero # | 2x + 6y = 12 $-x + 2y = 9$ | $\begin{bmatrix} 2 & 6 & 12 \\ -1 & 2 & 9 \end{bmatrix}$ |
| | 2x + 6y = 12 $-2x + 4y = 18$ | $ \begin{bmatrix} 2 \cdot r_2 \to R_2 \\ 2 & 6 & 12 \\ -2 & 4 & 18 \end{bmatrix} $ |
| ADD a nonzero multiple of a row to another | 2x + 6y = 12 $-2x + 4y = 18$ | $\begin{bmatrix} 2 & 6 & 12 \\ -2 & 4 & 18 \end{bmatrix}$ |
| | 10y = 30 | $\begin{bmatrix} r_2 + r_1 \to R_2 \\ 2 & 6 & 12 \\ 0 & 10 & 30 \end{bmatrix}$ |

Note: This method of solving systems is called **Gaussian Elimination**.

PRACTICE Solve the system of equations by using row operations to write a matrix in row-echelon form.

$$x + 3y + 4z = 2$$
$$2x + 5y + 7z = 9$$
$$4x + 8y + 10z = 14$$

| Matrix Row Operations | | |
|--|------------------------------|---|
| Row Operation | Sys of Eqn's | Matrix |
| SWAP two rows | -x + 2y = 9 $2x + 6y = 12$ | $\begin{bmatrix} -1 & 2 & 9 \\ 2 & 6 & 12 \end{bmatrix}$ |
| SWAP two rows | 2x + 6y = 12 $-x + 2y = 9$ | $\begin{bmatrix} 2 & 6 & 12 \\ -1 & 2 & 9 \end{bmatrix}$ |
| | 2x + 6y = 12 $-x + 2y = 9$ | $\begin{bmatrix} 2 & 6 & 12 \\ -1 & 2 & 9 \end{bmatrix}$ |
| MULTIPLY row by nonzero # | | $2 \cdot r_2 \rightarrow R_2$ |
| | 2x + 6y = 12 $-2x + 4y = 18$ | $\begin{bmatrix} 2 & 6 & 12 \\ -2 & 4 & 18 \end{bmatrix}$ |
| ADD a nonzero multiple of a row to another | 2x + 6y = 12 $-2x + 4y = 18$ | $\begin{bmatrix} 2 & 6 & 12 \\ -2 & 4 & 18 \end{bmatrix}$ |
| | | $r_2 + r_1 \rightarrow R_2$ |
| | 10y = 30 | $\begin{bmatrix} 2 & 6 & 12 \\ 0 & 10 & 30 \end{bmatrix}$ |

PRACTICE Solve the system of equations by using row operations to write a matrix in row-echelon form.

$$2x + 4y + 6z = 24$$
$$x + 5y + 12z = 60$$
$$3x + 6y + 15z = 20$$

| Matrix Row Operations | | |
|--|------------------------------|---|
| Row Operation | Sys of Eqn's | Matrix |
| SWAP two rows | -x + 2y = 9 $2x + 6y = 12$ | $\begin{bmatrix} -1 & 2 & 9 \\ 2 & 6 & 12 \end{bmatrix}$ |
| | 2x + 6y = 12 $-x + 2y = 9$ | $\begin{bmatrix} 2 & 6 & 12 \\ -1 & 2 & 9 \end{bmatrix}$ |
| MIII TIDI V | 2x + 6y = 12 $-x + 2y = 9$ | |
| MULTIPLY row by nonzero # | 2 | $2 \cdot r_2 \rightarrow R_2$ |
| | 2x + 6y = 12 $-2x + 4y = 18$ | $\begin{bmatrix} 2 & 6 & 12 \\ -2 & 4 & 18 \end{bmatrix}$ |
| ADD a nonzero multiple of a row to another | 2x + 6y = 12 $-2x + 4y = 18$ | $\begin{bmatrix} 2 & 6 & 12 \\ -2 & 4 & 18 \end{bmatrix}$ |
| | | $\begin{array}{c} r_2 + r_1 \rightarrow R_2 \\ \end{array}$ |
| | 10y = 30 | $\begin{bmatrix} 2 & 6 & 12 \\ 0 & 10 & 30 \end{bmatrix}$ |

<u>Solving a System of Equations Using Reduced Row – Echelon Form</u>

◆ Perform more Row Operations to get a more simplified matrix that's even easier to solve!

| Recall | "Row – Echelon" Form | New "Reduced Row – Echelon" Form |
|--|--|--|
| $\begin{bmatrix} 1 & \# \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ | # # # # $x + #y + #z = #$ # # $y + #z = #$ 1's ALONG diagonal 0's UNDER diagonal | $\begin{bmatrix} 1 & 0 & 0 & \# \\ 0 & 1 & 0 & \# \\ 0 & 0 & 1 & \# \end{bmatrix} \qquad \Rightarrow \qquad \begin{array}{c} x & = \# \\ y & = \# \\ z & = \# \end{array}$ 1's ALONG diagonal 0's UNDER & diagonal |

EXAMPLE

Solve the system of equations by writing a matrix in row-echelon form.

ADD
$$r_{2} + 1 \cdot r_{1} \rightarrow R_{2}$$

MULTIPLY
$$\frac{1}{9} \cdot r_{2} \rightarrow R_{2}$$

$$\begin{bmatrix} 1 & 7 & | 14 \\ 0 & 9 & | 18 \end{bmatrix}$$

$$\downarrow convert$$

$$x + 7y = 14$$

$$y = 2$$

$$x + 7(2) = 14$$

$$x + 14 = 14$$

$$x = 0$$

[LESS | MORE] work with matrices [LESS | MORE] work with equations

Note: This method is called *Gaussian* Elimination.

Solve the system of equations by writing a matrix in *reduced* row-echelon form.

[LESS | MORE] work with matrices [LESS | MORE] work with equations

Note: This method is called *Gauss-Jordan* Elimination.

EXAMPLE Solve the system of equations by using row operations to write a matrix in *REDUCED* row-echelon form.

$$4x + 2y + 3z = 6$$
$$x + y + z = 3$$
$$5x + y + 2z = 5$$

| Matrix Row Operations | | |
|------------------------------|------------------------------|---|
| Row Operation | Sys of Eqn's | Matrix |
| CWAD two rows | -x + 2y = 9 $2x + 6y = 12$ | $\begin{bmatrix} -1 & 2 & 9 \\ 2 & 6 & 12 \end{bmatrix}$ |
| SWAP two rows | 2x + 6y = 12 $-x + 2y = 9$ | $\begin{bmatrix} 2 & 6 & 12 \\ -1 & 2 & 9 \end{bmatrix}$ |
| MIII TIDI V row by | 2x + 6y = 12 $-x + 2y = 9$ | |
| MULTIPLY row by nonzero # | 2x + 6y = 12 | $2 \cdot r_2 \to R_2$ |
| | 2x + 6y = 12 $-2x + 4y = 18$ | $\begin{bmatrix} 2 & 0 & 12 \\ -2 & 4 & 18 \end{bmatrix}$ |
| ADD a nonzero | 2x + 6y = 12 $-2x + 4y = 18$ | $\begin{bmatrix} 2 & 6 & 12 \\ -2 & 4 & 18 \end{bmatrix}$ |
| multiple of a row to another | | $r_2 + r_1 \rightarrow R_2$ |
| | 10y = 30 | $\begin{bmatrix} 2 & 6 & 12 \\ 0 & 10 & 30 \end{bmatrix}$ |