TOPIC: INTRODUCTION TO MATRICES
Introduction to Matrices

- A **Matrix** is a way to ______________ numbers into rows & columns.
  - An **Augmented Matrix** represents a system of linear equations.

Recall: System of Equations	New: Matrix
(2 coefficients + 1 constant) 3 numbers	____ columns
2 equations $\begin{cases} -7x + (-3y) = -4 \\ 2x + \frac{1}{2}y = 13 \end{cases}$	____ rows $\left[\begin{array}{cc\|c} -7 & -3 & -4 \\ 2 & \frac{1}{2} & 13 \end{array}\right]$
	| means " "
	____ × ____ matrix

EXAMPLE Represent the system of equations using an augmented matrix.

$$2x + (-3y) + z = -4$$

$$6x + 3y = 13$$

$$y - z = 8$$

$$\left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}\right]$$

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PRACTICE Write the equations in standard form, then represent the system using an augmented matrix.

$$3x + 5y - 9 = 0$$
$$8x = -4y + 3$$

PRACTICE Write the system of equations represented by the augmented matrix shown.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 5 & 4 & 1 \\ 4 & 7 & 0 & 12 \end{array}\right]$$

TOPIC: INTRODUCTION TO MATRICES
Performing Row Operations on Matrices

◆ Just as we did *operations* to eqn's when solving **Systems of Eqn's**, we'll do *operations* on the _______ of a matrix.

New **Matrix Row Operations**

<table>
<tr><th>Row Operation</th><th>System of Equations</th><th>Matrix</th><th>Notation</th></tr>
<tr><td>SWAP two rows</td><td>$-x+2y=9$
$2x+6y=12$

$2x+6y=12$
$-x+2y=9$</td><td>$\left[\begin{array}{cc|c} -1 & 2 & 9 \\ 2 & 6 & 12 \end{array}\right]$

$\left[\begin{array}{cc|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$</td><td>Old row New row
$r_1 \leftrightarrow R_2$</td></tr>
<tr><td>MULTIPLY one row by any nonzero #</td><td>$2x+6y=12$
$-x+2y=9$

$2x+6y=12$
$-2x+4y=18$</td><td>$\left[\begin{array}{cc|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$

$\left[\begin{array}{cc|c} 2 & 6 & 12 \\ -2 & 4 & 18 \end{array}\right]$</td><td>$2 \cdot r_2 \rightarrow R_2$
(Only affects row 2)</td></tr>
<tr><td>ADD a nonzero multiple of one row to another</td><td>$2x+6y=12$
$-2x+4y=18$

$10y=30$</td><td>$\left[\begin{array}{cc|c} 2 & 6 & 12 \\ -2 & 4 & 18 \end{array}\right]$

$\left[\begin{array}{cc|c} 2 & 6 & 12 \\ 0 & 10 & 30 \end{array}\right]$</td><td>$r_2 + 1 \cdot r_1 \rightarrow R_2$
(Only affects row 2)</td></tr>
</table>

EXAMPLE Perform the given row operation on the augmented matrix and write the new matrix.

$$\left[\begin{array}{ccc|c} 2 & -6 & 4 & 10 \\ 3 & 8 & -7 & 0 \\ -1 & 5 & 9 & 3 \end{array}\right]$$

(A) $r_2 \leftrightarrow R_3$

(B) $\frac{1}{2} \cdot r_1 \rightarrow R_1$

(C) $r_2 + 3 \cdot r_3 \rightarrow R_2$

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PRACTICE Perform the indicated **Row Operation**.

$$\left[\begin{array}{ccc|c} 7 & 9 & 3 & 4 \\ 1 & 8 & 4 & 3 \\ 2 & 0 & 5 & 2 \end{array}\right]$$

SWAP $R_1 \leftrightarrow R_2$

PRACTICE Perform the indicated **Row Operation**.

$$\left[\begin{array}{ccc|c} 2 & 5 & 4 & 3 \\ 5 & 3 & 1 & 2 \\ 1 & 2 & 3 & 3 \end{array}\right]$$

ADD $R_1 + 2 \cdot R_3 \rightarrow R_1$

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Solving a System of Equations Using Matrices (Row – Echelon Form)

◆ You may have to solve a system of equations expressed as a matrix.

- Use **Row Operations** to obtain a matrix with 1's ________ the diagonal & 0's ________ the diagonal.

Given

$$-x + 2y = 4$$
$$x + 7y = 14$$

OR

$$\left[\begin{array}{cc|c} -1 & 2 & 4 \\ 1 & 7 & 14 \end{array}\right]$$

$\xrightarrow{Row\ Op's}$

"Row-Echelon" Form

$$\left[\begin{array}{cc|c} 1 & \# & \# \\ 0 & 1 & \# \end{array}\right]$$

(ANY number)

OR

$$x + \#y = \#$$
$$y = \#$$

◆ Tips for Solving:

1) Work row-by-row from top-to-bottom

2) Whenever you get a 1 along the diagonal, get everything under it to be 0 *BEFORE* trying to get the next 1

EXAMPLE Solve the system of equations using matrices and row operations.

$$-x + 2y = 4$$
$$x + 7y = 14$$

Matrix Row Operations

Row Operation	Sys of Eqn's	Matrix
SWAP two rows	$-x + 2y = 9$ $2x + 6y = 12$ $2x + 6y = 12$ $-x + 2y = 9$	$\left[\begin{array}{cc\|c} -1 & 2 & 9 \\ 2 & 6 & 12 \end{array}\right]$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$
MULTIPLY row by nonzero #	$2x + 6y = 12$ $-x + 2y = 9$ $2x + 6y = 12$ $-2x + 4y = 18$	$\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$ $2 \cdot r_2 \rightarrow R_2$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -2 & 4 & 18 \end{array}\right]$
ADD a nonzero *multiple* of a row to another	$2x + 6y = 12$ $-2x + 4y = 18$ $10y = 30$	$\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -2 & 4 & 18 \end{array}\right]$ $r_2 + r_1 \rightarrow R_2$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ 0 & 10 & 30 \end{array}\right]$

Note: This method of solving systems is called **Gaussian Elimination.**

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PRACTICE Solve the system of equations by using row operations to write a matrix in row-echelon form.

$$x + 3y + 4z = 2$$
$$2x + 5y + 7z = 9$$
$$4x + 8y + 10z = 14$$

Matrix Row Operations

Row Operation	Sys of Eqn's	Matrix
SWAP two rows	$-x + 2y = 9$ $2x + 6y = 12$ $2x + 6y = 12$ $-x + 2y = 9$	$\left[\begin{array}{cc\|c} -1 & 2 & 9 \\ 2 & 6 & 12 \end{array}\right]$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$
MULTIPLY row by nonzero #	$2x + 6y = 12$ $-x + 2y = 9$ $2x + 6y = 12$ $-2x + 4y = 18$	$\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$ $2 \cdot r_2 \rightarrow R_2$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -2 & 4 & 18 \end{array}\right]$
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PRACTICE Solve the system of equations by using row operations to write a matrix in row-echelon form.

$$2x + 4y + 6z = 24$$
$$x + 5y + 12z = 60$$
$$3x + 6y + 15z = 20$$

Matrix Row Operations

Row Operation	Sys of Eqn's	Matrix
SWAP two rows	$-x + 2y = 9$ $2x + 6y = 12$ $2x + 6y = 12$ $-x + 2y = 9$	$\left[\begin{array}{cc\|c} -1 & 2 & 9 \\ 2 & 6 & 12 \end{array}\right]$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$
MULTIPLY row by nonzero #	$2x + 6y = 12$ $-x + 2y = 9$ $2x + 6y = 12$ $-2x + 4y = 18$	$\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$ $2 \cdot r_2 \rightarrow R_2$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -2 & 4 & 18 \end{array}\right]$
ADD a nonzero *multiple* of a row to another	$2x + 6y = 12$ $-2x + 4y = 18$ $10y = 30$	$\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -2 & 4 & 18 \end{array}\right]$ $r_2 + r_1 \rightarrow R_2$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ 0 & 10 & 30 \end{array}\right]$

TOPIC: INTRODUCTION TO MATRICES
Solving a System of Equations Using *Reduced* Row – Echelon Form

◆ Perform more **Row Operations** to get a more simplified matrix that's even easier to solve!

Recall "Row – Echelon" Form	New "*Reduced* Row – Echelon" Form
$\left[\begin{array}{ccc\|c} 1 & \# & \# & \# \\ 0 & 1 & \# & \# \\ 0 & 0 & 1 & \# \end{array}\right] \Rightarrow \begin{aligned} x + \#y + \#z &= \# \\ y + \#z &= \# \\ z &= \# \end{aligned}$	$\left[\begin{array}{ccc\|c} 1 & 0 & 0 & \# \\ 0 & 1 & 0 & \# \\ 0 & 0 & 1 & \# \end{array}\right] \Rightarrow \begin{aligned} x &= \# \\ y &= \# \\ z &= \# \end{aligned}$
1's **ALONG** diagonal 0's **UNDER** diagonal	1's **ALONG** diagonal 0's **UNDER** & ________ diagonal

EXAMPLE

Solve the system of equations by writing a matrix in row-echelon form.

$$\left[\begin{array}{cc|c} 1 & 7 & 14 \\ -1 & 2 & 4 \end{array}\right]$$

ADD $r_2 + 1 \cdot r_1 \to R_2$

$$\left[\begin{array}{cc|c} 1 & 7 & 14 \\ 0 & 9 & 18 \end{array}\right]$$

MULTIPLY $\frac{1}{9} \cdot r_2 \to R_2$

$$\left[\begin{array}{cc|c} 1 & 7 & 14 \\ 0 & 1 & 2 \end{array}\right]$$

↓ *convert*

$$x + 7y = 14$$
$$y = 2$$
$$x + 7(2) = 14$$
$$x + 14 = 14$$
$$x = 0$$

[LESS | MORE] work with matrices
[LESS | MORE] work with equations

Note: This method is called ***Gaussian*** Elimination.

Solve the system of equations by writing a matrix in *reduced* row-echelon form.

$$\left[\begin{array}{cc|c} 1 & 7 & 14 \\ -1 & 2 & 4 \end{array}\right]$$

ADD $r_2 + 1 \cdot r_1 \to R_2$

$$\left[\begin{array}{cc|c} 1 & 7 & 14 \\ 0 & 9 & 18 \end{array}\right]$$

MULTIPLY $\frac{1}{9} \cdot r_2 \to R_2$

$$\left[\begin{array}{cc|c} 1 & 7 & 14 \\ 0 & 1 & 2 \end{array}\right]$$

↓

$$\left[\begin{array}{cc|c} \quad & \quad & \quad \\ \quad & \quad & \quad \end{array}\right]$$

↓ *convert*

__ x + __ y = __
__ x + __ y = __

[LESS | MORE] work with matrices
[LESS | MORE] work with equations

Note: This method is called ***Gauss-Jordan*** Elimination.

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EXAMPLE Solve the system of equations by using row operations to write a matrix in ***REDUCED*** row-echelon form.

$$4x + 2y + 3z = 6$$
$$x + y + z = 3$$
$$5x + y + 2z = 5$$

Matrix Row Operations

Row Operation	Sys of Eqn's	Matrix
SWAP two rows	$-x + 2y = 9$ $2x + 6y = 12$ $2x + 6y = 12$ $-x + 2y = 9$	$\left[\begin{array}{cc\|c} -1 & 2 & 9 \\ 2 & 6 & 12 \end{array}\right]$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$
MULTIPLY row by nonzero #	$2x + 6y = 12$ $-x + 2y = 9$ $2x + 6y = 12$ $-2x + 4y = 18$	$\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -1 & 2 & 9 \end{array}\right]$ $2 \cdot r_2 \rightarrow R_2$ $\left[\begin{array}{cc\|c} 2 & 6 & 12 \\ -2 & 4 & 18 \end{array}\right]$
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