

TOPIC: SOLVING LINEAR EQUATIONS

Introduction to Linear Equations

◆ Recall: An **equation** is a statement that two algebraic **expressions** are equal.

New **Linear Equations in One Variable**

expression
 $2x + 6$

expression
 0

$ax + b = c$

a, b, c are _____ numbers

a _____ 0

◆ The **solution** to an equation is the value of the variable that makes the statement _____ when plugged in.

EXAMPLE

Determine whether the given value is a solution to the equation.

(A)

$$2x + 6 = 0; x = -3$$

(B)

$$5 = 8w - 3; w = -1$$

◆ For a linear equation, write the **solution set** by putting the solution in set brackets { }.

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EXAMPLE

Determine whether each of the following is a linear equation in one variable.

(A) $4x - 7 + 3$

(B) $5y - y = 2$

(C) $6x + 1 = 2t^2$

PRACTICE

Identify the following as either an expression or equation.

(A) $\frac{2m}{3} + 8$

(B) $4(a - 2) = 21$

[EXPRESSION | EQUATION]

[EXPRESSION | EQUATION]

PRACTICE

Which of the following is a linear equation in one variable?

A. $x + 5 = 12$

B. $x^2 = 25$

C. $y + z = 10$

D. $x - 3 < 7$

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PRACTICE

Which of the following is a linear equation in one variable?

A. $2(x + 5) - 3x = x^2 + 1$

B. $x(x + 4) = 0$

C. $4(x + 3) = 2x + 18$

D. $x^2 + 5x = 10$

PRACTICE

Verify that the given value is a solution to the equation.

(A) $y = -2; 5y + 4 = 14$

[SOLUTION | NOT A SOLUTION]

(B) $a = 2; 4a + 3 = 2a + 9$

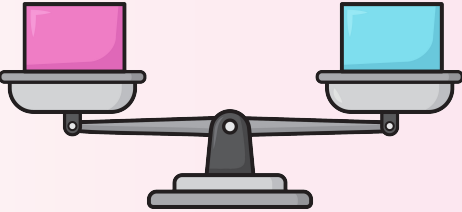
[SOLUTION | NOT A SOLUTION]

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Addition and Subtraction Properties of Equality

- ◆ To **solve** an equation, _____ the variable using operations like **addition** and **subtraction**.
- ▶ Operations must **ALWAYS** be done to _____ sides of an equation to create *equivalent equations*.

Addition Property of Equality	Subtraction Property of Equality
<p>If $a = b$, then $a \underline{\quad} = b \underline{\quad}$</p> <p>Use when eqn has [ADDITION SUBTRACTION]</p> <p>$x - 6 = 0$ $x - 6 \underline{\quad} = 0 \underline{\quad}$ $\underline{\quad} = \underline{\quad}$</p>	<p>If $a = b$, then $a \underline{\quad} = b \underline{\quad}$</p> <p>Use when eqn has [ADDITION SUBTRACTION]</p> <p>$0 = x + 2$ $0 \underline{\quad} = x + 2 \underline{\quad}$ $\underline{\quad} = \underline{\quad}$</p>



- ◆ Check your solution by replacing variable in original equation & verifying that it makes the statement **true**.

EXAMPLE

Solve the linear equation, then check your solution.

$$y - 1.2 = 5.8$$

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PRACTICE

Solve the given linear equation using addition and subtraction properties of equality.

(A) $x + \frac{2}{8} = -\frac{3}{8}$

(B) $-5.4 + c = 1.6$

PRACTICE

Solve the given linear equation using addition and subtraction properties of equality.

(A) $6h - (-12) = 5 + 5h$

(B) $2(x + 5) = 3(x - 1)$

(C) $3(y + 3) + (1 - y) = 3y + 14$

EXAMPLE

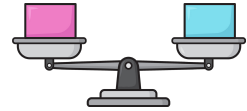
Translate the following statement into a linear equation and solve.

A number decreased by 7 is equal to 15. What is the number?

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Multiplication and Division Properties of Equality

◆ Recall: Solve a linear equation by using operations done to **both** sides to isolate the variable.



► **Multiplication** and **division** can also be used to create *equivalent equations*.

Multiplication Property of Equality	Division Property of Equality
<div data-bbox="349 541 652 646" style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">If $a = b$, then $a \underline{\quad} = b \underline{\quad}$</div> <p data-bbox="217 682 784 716">Use when eqn has [MULTIPLICATION DIVISION]</p> $\frac{x}{2} = 9$ $\frac{x}{2} \underline{\quad} = 9 \underline{\quad}$ $\underline{\quad} = \underline{\quad}$	<div data-bbox="969 541 1273 646" style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">If $a = b$, then $a = b$</div> <p data-bbox="833 682 1399 716">Use when eqn has [MULTIPLICATION DIVISION]</p> $20 = 5x$ $20 = 5x$ $\underline{\quad} = \underline{\quad}$

◆ Isolate the variable *term* using _____ BEFORE using \times/\div to fully isolate *variable*.

EXAMPLE Solve the linear equation, then check your solution.

$$3a - 4 = 11$$

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EXAMPLE

Solve the linear equation, then check your solution.

$$1.2a + 2.3 = 5.9$$

PRACTICE

Solve the given linear equation using multiplication and division properties of equality.

(A)

$$-8x = 64$$

(B)

$$\frac{y}{4} = -\frac{21}{6}$$

(C)

$$\frac{126}{14} = 3y$$

PRACTICE

Solve the given linear equation, then check your answer.

(A)

$$-(h + 3) = 11$$

(B)

$$0.5t + 1.5t = 7 + 3 - 4$$

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EXAMPLE

Translate the following statement into a linear equation and solve.

Three times L equals 54. What is the value of L ?

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Multiplication Property for Fraction Coefficients

◆ If the variable in a linear equation has a fraction **coefficient**, cancel by multiplying both sides by its _____.

▶ Recall: The product of a **number** and its **reciprocal** is 1. For example, $\frac{2}{3} \cdot \frac{3}{2} = 1$

EXAMPLE

Solve the following equations.

(A)

$$\frac{3}{4}x = 9$$

(B)

$$10 = \frac{5}{3}y$$

Recall

If $a = b$,
then $ac = bc$

PRACTICE

Solve the given linear equation, then check your answer.

$$\frac{1}{2} = \frac{3}{4}x$$

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Name	Use when equation has...	Property of Equality	Example		
		If $a = b$, then...			
Addition	-	$a + c = b + c$	$\begin{aligned}x - 6 &= 0 \\x - 6 + 6 &= 0 + 6 \\x &= 6\end{aligned}$		
Subtraction	+	$a - c = b - c$	$\begin{aligned}0 &= x + 2 \\0 - 2 &= x + 2 - 2 \\-2 &= x\end{aligned}$		
Multiplication	\div	$ac = bc$	<table border="1"><tr><td>$\begin{aligned}\frac{x}{2} &= 24 \\2 \cdot \frac{x}{2} &= 24 \cdot 2 \\x &= 48\end{aligned}$</td><td>$\begin{aligned}\frac{3}{4}x &= 9 \\\frac{4}{3} \cdot \frac{3}{4}x &= 9 \cdot \frac{4}{3} \\x &= 12\end{aligned}$</td></tr></table>	$\begin{aligned}\frac{x}{2} &= 24 \\2 \cdot \frac{x}{2} &= 24 \cdot 2 \\x &= 48\end{aligned}$	$\begin{aligned}\frac{3}{4}x &= 9 \\\frac{4}{3} \cdot \frac{3}{4}x &= 9 \cdot \frac{4}{3} \\x &= 12\end{aligned}$
$\begin{aligned}\frac{x}{2} &= 24 \\2 \cdot \frac{x}{2} &= 24 \cdot 2 \\x &= 48\end{aligned}$	$\begin{aligned}\frac{3}{4}x &= 9 \\\frac{4}{3} \cdot \frac{3}{4}x &= 9 \cdot \frac{4}{3} \\x &= 12\end{aligned}$				
Division	\times	$\frac{a}{c} = \frac{b}{c}$	$\begin{aligned}20 &= 5x \\\frac{20}{5} &= \frac{5x}{5} \\4 &= x\end{aligned}$		

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Strategy for Solving Linear Equations

◆ To solve **ANY** linear equation, *simplify* & then use *multiple* properties of equality. You can follow these steps:

EXAMPLE

Solve the linear equation.

$$3(x - 2) + 2 = x + 8$$

Recall

If $a = b$, then...

$$a + c = b + c \quad | \quad ac = bc$$

$$a - c = b - c \quad | \quad \frac{a}{c} = \frac{b}{c}$$

(Properties of Equality)

HOW TO: Solve Linear Equations

- 1) Simplify** both sides of the equation
 - *Distribute* into ()
 - *Combine* like terms
- 2) Use** _____ props. to **collect**:
 - All *variable* terms on one side
 - All *constant* terms on other side
- 3) Use** _____ props. to **isolate** variable
- 4) Check** solution by plugging in *orig. eqn*

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PRACTICE

Solve the given linear equation. Check your solution.

(A) $2(x + 3) = 14$

(B) $-5 - y = 3(y + 9)$

HOW TO: Solve Linear Equations

- 1) Simplify** both sides of the equation
 - *Distribute* into ()
 - *Combine* like terms
- 2) Use +/− props. to collect:**
 - All *variable* terms on one side
 - All *constant* terms on other side
- 3) Use ×/÷ props. to isolate** variable
- 4) Check** solution by plugging in *orig. eqn*

PRACTICE

Solve the given linear equation. Check your solution.

$$4(x + 1) - 3(x - 2) = 2x + 5$$

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Solving Linear Equations with Fraction or Decimal Coefficients

- ◆ Some linear equations have **fractions** or **decimals** that should be _____ before following the steps for solving.
 - ▶ For equations with **fractions**, _____ both sides by the **Least Common Denominator** of all fractions.
 - ▶ For equations with **decimals**, multiply both sides by powers of 10 to move the _____ to the right.

EXAMPLE

Solve the linear equation.

(A)

$$\frac{1}{3}x - \frac{5}{6} = \frac{1}{2}$$

(B)

$$0.36 = 0.03x + 0.3$$

HOW TO: Solve Linear Equations

0) Clear Fractions or Decimals

- Multiply by _____ or power of _____

1) Simplify both sides of the equation

2) Use +/− props. to **collect**:

- All *variable* terms on one side
- All *constant* terms on other side

3) Use ×/÷ props. to **isolate** variable

4) **Check** solution by plugging in *orig. eqn*

- ◆ Multiplying by 10^1 moves the decimal right _____, multiplying by 10^2 moves the decimal right _____, etc.

PRACTICE

Solve the following equations with fractions.

(A)

$$\frac{x}{4} - \frac{2}{3} = \frac{x}{6}$$

(B)

$$\frac{2}{5}y - 3 = \frac{1}{4}$$

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EXAMPLE

Solve the following equation.

$$\frac{2x - 1}{5} + \frac{3x + 2}{9} = x + 1$$

HOW TO: Solve Linear Equations

0) Clear Fractions or Decimals

- Multiply by **LCD** or power of **10**

1) Simplify both sides of the equation

2) Use +/− props. to **collect**:

- All *variable* terms on one side
- All *constant* terms on other side

3) Use ×/÷ props. to **isolate** variable

4) **Check** solution by plugging in *orig. eqn*

EXAMPLE

Solve the following equation.

$$\frac{x}{3} + \frac{10}{15} = \frac{4x}{6}$$

PRACTICE

Solve the following equations with decimals.

(A)

$$4.1x + 2.4 = 3.6x - (-4)$$

(B)

$$1.5(z - 2) + 0.6 = 0.3z + 9$$

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Classifying Linear Equations

◆ Recall: The solution set of a linear eq. is the set of all values of x that make a true statement when plugged in.

► Linear equations can be classified by the ___ of solutions in their solution set.

New Categories of Linear Equations		
$2x + 4 = 14$ $\quad \cancel{-4} \quad \cancel{-4}$ $\quad \frac{\cancel{2}x}{2} = \frac{10}{2}$ $\quad x = 5$	$3x + 2 + x = 4x + 2$	$7x + 3 = 7x$
<ul style="list-style-type: none">• Conditionally _____• Exactly _____ solution Solution set:	<ul style="list-style-type: none">• Always _____• _____ solutions Solution set: $\{x x \quad \}$	<ul style="list-style-type: none">• Always _____• _____ solutions Solution set:

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PRACTICE

How many solutions do the following equations have?

(A) $4(x - 3) = 4x - 12$

[ONE SOLUTION | INFINITE SOLUTIONS | NO SOLUTION]

(B) $\frac{1}{2}(x + 4) - \frac{1}{3} = \frac{1}{6}x - 2$

[ONE SOLUTION | INFINITE SOLUTIONS | NO SOLUTION]

(C) $0.25x + 0.6 = 0.25(x + 3)$

[ONE SOLUTION | INFINITE SOLUTIONS | NO SOLUTION]

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PRACTICE

Classify each of the following equations.

(A) $12(x - 4) = 4(3x + 12)$

[**CONDITIONAL | IDENTITY | CONTRADICTION**]

(B) $6(x - 1) + 13 - x = 5x + 7$

[**CONDITIONAL | IDENTITY | CONTRADICTION**]

(C) $2.5x + 3.1 = 1.2(x - 2)$

[**CONDITIONAL | IDENTITY | CONTRADICTION**]