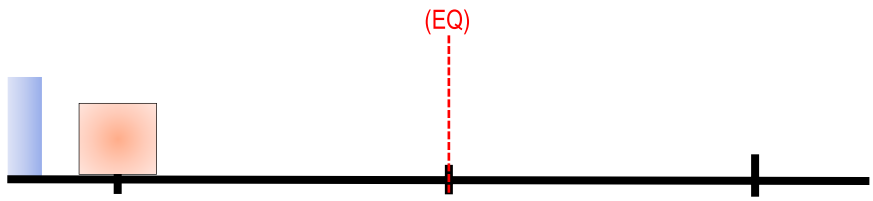
**CONCEPT: Intro to Simple Harmonic Motion**

● The most common type of Simple Harmonic Motion (aka Oscillation) is the mass-spring system.




| | | |
|---|---|---|
| **x = ______ = ______** | **x = ______** | **x = ______ = ______** |
| **v = ______** | **v = ______** | **v = ______** |
| **F = ______** | **F = ______** | **F = ______** |
| **a = ______** | **a = ______** | **a = ______** |

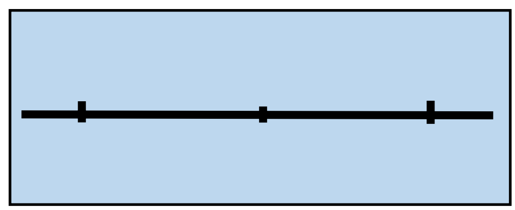
● Amplitude → A:
- ____________ displacement, |**x**|
- always the _________________.

● Period → T [seconds/cycle]
- Time for one ____________ cycle.

● Frequency → $f$ = 1/T [cycles/second]

● Angular frequency → ω [rad/second]
- ω = _______ = _______

EXAMPLE 1: A mass on a spring is pulled 1m away from its equilibrium position, then released from rest. The mass takes 2s to reach maximum displacement on the other side. Calculate the **(a)** amplitude, **(b)** period, **(c)** angular frequency of the motion.



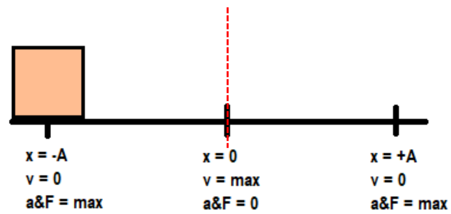
<u>PRACTICE:</u> A mass-spring system with an angular frequency ω = 8π rad/s oscillates back and forth**.** (**a**) Assuming it starts from rest, how much time passes before the mass has a speed of 0 again? (**b**) How many full cycles does the system complete in 60s?

| **Mass-Spring SHM Equations** |
| --- |
| $\lvert F_S \rvert = \lvert F_A \rvert = kx$<br>$a = -\frac{k}{m}x$ |
| $\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$<br>$N \text{ [cycles]} = \frac{t \text{ [time]}}{T \text{ [Period]}} = t * f$ |

## CONCEPT: Equations for Simple Harmonic Motion

● In Simple Harmonic Motion, acceleration <u>NOT</u> constant → kinematic equations?

| Old Equations → x (position) | |
|---|---|
| $F_s = -k\,x$ | → $F_{max}$ = _____ |
| $a = -\frac{k\,x}{m}$ | → $a_{max}$ = _____ |
| **New Equations → t (time)** | |
| $x(t) = +\ A\cos(\omega t)$ | → $x_{max}$ = _____ |
| $v(t) = -\ A\omega\ \sin(\omega t)$ | → $v_{max}$ = _____ |
| $a(t) = -A\omega^2\cos(\omega t)$ | → $a_{max}$ = _____ |

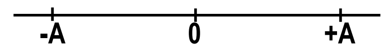



- Calculator must be in _________.

● Combining $a_{max}(x)$ and $a_{max}(t)$ → $\boxed{\omega = 2\pi f = \frac{2\pi}{T} = \_\_\_\_\_\_\_}$

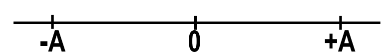
<u>EXAMPLE 1</u>: A 4-kg mass is attached to a spring where k = 200[N/m]. The mass is pulled 2m and released from rest. Find the (**a**) angular frequency, (**b**) velocity 0.5s after release, (**c**) acceleration when x = 0.5m, and (**d**) the period of oscillation.

PRACTICE: A 4-kg mass on a spring is released 5 m away from equilibrium position and takes 1.5 s to reach its equilibrium position. (**a**) Find the spring's force constant. (**b**) Find the object's max speed.




| Mass-Spring SHM Equations | |
|---|---|
| $\lvert F_S \rvert = \lvert F_A \rvert = kx$ | $\rightarrow F_{max} = \pm kA$ |
| $a = -\frac{k}{m}x$ | $\rightarrow a_{max} = \pm \frac{k}{m}A$ |
| $x(t) = +\ A\cos(\omega t)$ | $\rightarrow x_{max} = \pm A$ |
| $v(t) = -A\omega\ \sin(\omega t)$ | $\rightarrow v_{max} = \pm A\omega$ |
| $a(t) = -A\omega^2\cos(\omega t)$ | $\rightarrow a_{max} = \pm A\omega^2$ |
| $\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$ | |
| $N\ [\text{cycles}] = \frac{t\ [\text{time}]}{T\ [\text{Period}]} = t * f$ | |

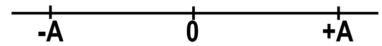
EXAMPLE: A 4-kg mass is attached to a horizontal spring and oscillates at 2 Hz. If mass is moving with 10 m/s when it crosses its equilibrium position, (**a**) how long does it take to get from equilibrium to its max distance? Find the (**b**) amplitude and (**c**) maximum acceleration.





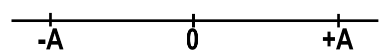
| Mass-Spring SHM Equations | |
|---|---|
| $\lvert F_S \rvert = \lvert F_A \rvert = kx$ | $\rightarrow F_{max} = \pm kA$ |
| $a = -\frac{k}{m}x$ | $\rightarrow a_{max} = \pm \frac{k}{m}A$ |
| $x(t) = +\ A\cos(\omega t)$ | $\rightarrow x_{max} = \pm A$ |
| $v(t) = -A\omega\ \sin(\omega t)$ | $\rightarrow v_{max} = \pm A\omega$ |
| $a(t) = -A\omega^2\cos(\omega t)$ | $\rightarrow a_{max} = \pm A\omega^2$ |
| $\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$ | |
| $N\ [\text{cycles}] = \frac{t\ [\text{time}]}{T\ [\text{Period}]} = t * f$ | |

PRACTICE: What is the equation for the position of a mass moving on the end of a spring which is stretched 8.8cm from equilibrium and then released from rest, and whose period is 0.66s? What will be the object's position after 1.4s?




| Mass-Spring SHM Equations | |
|---|---|
| $\lvert F_S \rvert = \lvert F_A \rvert = kx$ | $\rightarrow F_{max} = \pm kA$ |
| $a = -\frac{k}{m}x$ | $\rightarrow a_{max} = \pm\frac{k}{m}A$ |
| $x(t) = +\ A\cos(\omega t)$ | $\rightarrow x_{max} = \pm A$ |
| $v(t) = -A\omega\ \sin(\omega t)$ | $\rightarrow v_{max} = \pm A\omega$ |
| $a(t) = -A\omega^2\cos(\omega t)$ | $\rightarrow a_{max} = \pm A\omega^2$ |
| $\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$ | |
| $N\ [\text{cycles}] = \frac{t\ [\text{time}]}{T\ [\text{Period}]} = t * f$ | |

EXAMPLE: The velocity of a particle on a spring is given by the equation v(t) = -6.00 sin (3π t). Determine the (**a**) frequency of motion, (**b**) amplitude, and (**c**) velocity of the particle at t = 0.5s.





| Mass-Spring SHM Equations | |
|---|---|
| $\lvert F_S \rvert = \lvert F_A \rvert = kx$ | $\rightarrow F_{max} = \pm kA$ |
| $a = -\frac{k}{m}x$ | $\rightarrow a_{max} = \pm\frac{k}{m}A$ |
| $x(t) = +\ A\cos(\omega t)$ | $\rightarrow x_{max} = \pm A$ |
| $v(t) = -A\omega\ \sin(\omega t)$ | $\rightarrow v_{max} = \pm A\omega$ |
| $a(t) = -A\omega^2\cos(\omega t)$ | $\rightarrow a_{max} = \pm A\omega^2$ |
| $\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$ | |
| $N\ [\text{cycles}] = \frac{t\ [\text{time}]}{T\ [\text{Period}]} = t * f$ | |