

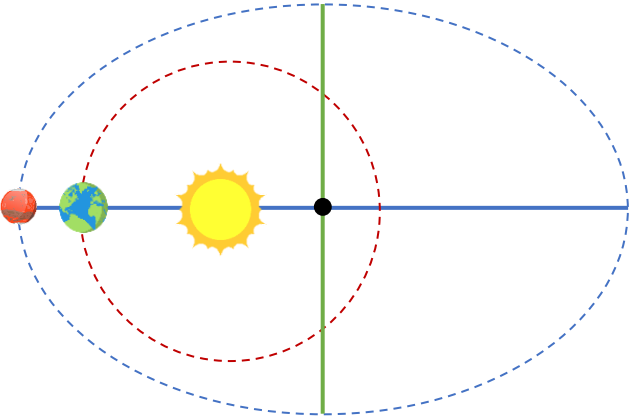
CONCEPT: Kepler’s Third Law for Elliptical Orbits

- Kepler’s 3rd Law also works for elliptical orbits! Instead of orbital distance r, we replace it with semi-major axis a.

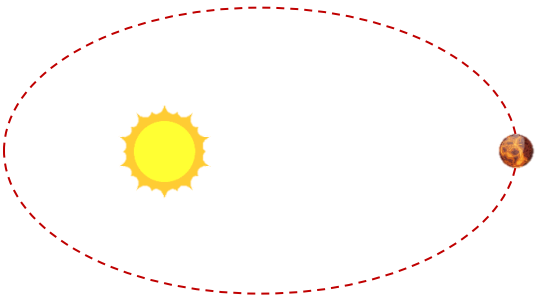
- For circular orbits, _____.

Circular Orbits
$T^2 = \frac{4\pi^2}{GM} r^3$

Elliptical Orbits
$T^2 = \frac{4\pi^2}{GM} a^3$



EXAMPLE: A planet orbits a star of mass 4×10^{30} kg in an elliptical orbit, and scientists have calculated its eccentricity to be 0.4. Its aphelion distance is 1.5×10^{11} m. What is the orbital period?



ELLIPTICAL ORBITS	CONSTANTS
$a = \frac{R_a + R_p}{2}$ $R_a = a(1 + e)$ $R_p = a(1 - e)$ $T^2 = \frac{4\pi^2 a^3}{GM}$	$G = 6.67\times10^{-11}$

PRACTICE: Comet Halley has a highly elliptical orbit around the Sun, circling once every 75.6 years with its closest point to the Sun being only 0.57AU (“Astronomical Unit”, where 1AU = 1.5×10^{11} m and represents the average Earth-Sun distance). How far will Comet Halley get from the Sun?

ELLIPTICAL ORBITS	CONSTANTS
$a = \frac{R_a + R_p}{2}$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$R_a = a(1 + e)$	$M_E = 5.97 \times 10^{24} \text{ kg}$
$R_p = a(1 - e)$	$R_E = 6.37 \times 10^6 \text{ m}$
$T^2 = \frac{4\pi^2 a^3}{GM}$	$M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$
	$R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$

EXAMPLE: A distant star is believed to be orbiting an extremely massive black hole. Careful measurements show this star moves in a highly elliptical orbit ($e = 0.9$) with a period of 14 years. Recently, this star reached its periapsis of 1.8×10^{13} m. What is the mass of the alleged black hole this star orbits?

ELLIPTICAL ORBITS	CONSTANTS
$a = \frac{R_a + R_p}{2}$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$R_a = a(1 + e)$	$M_E = 5.97 \times 10^{24} \text{ kg}$
$R_p = a(1 - e)$	$R_E = 6.37 \times 10^6 \text{ m}$
$T^2 = \frac{4\pi^2 a^3}{GM}$	$M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$
	$R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$