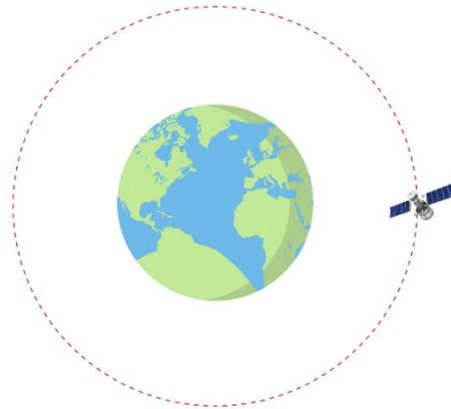


CONCEPT: Velocity of a Satellite

- For a satellite in circular orbit, the gravitational force keeps it in _____.

- The orbital speed & distance are related by:

$$v_{\text{sat}} = \underline{\hspace{2cm}}$$



- **M** = mass of planet
- **r** = orbital distance (not height!)
- For every value of **r**, there is an exact speed required to maintain circular orbit.

EXAMPLE: Calculate the height of the International Space Station, which orbits at 7,670 m/s in a nearly circular orbit.

EQUATIONS		CONSTANTS
$F_G = \frac{Gm_1m_2}{r^2}$	$r = R + h$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$ $M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$
$g_{\text{near}} = \frac{GM}{R^2}$	$g_{\text{far}} = \frac{GM}{r^2}$	
$v_{\text{sat}} = \sqrt{\frac{GM}{r}}$		

PRACTICE: Suppose that you used some geometry and kinematics to estimate that the Earth goes around the Sun with an orbital speed of approximately 30,000 m/s (60,000 mph), and that the Sun is approximately 150 million kilometers away from the Earth. Use this information to estimate the mass of the Sun.

EQUATIONS	CONSTANTS
$F_G = \frac{Gm_1m_2}{r^2} \quad r = R + h$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$g = \frac{GM}{r^2} \quad g_{\text{surf}} = \frac{GM}{R^2}$	$M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$
$v_{\text{sat}} = \sqrt{\frac{GM}{r}}$	

EXAMPLE: Two satellites are in circular orbits around a mysterious planet that is $8 \times 10^7 \text{ m}$ in diameter. The first satellite has mass 68kg, orbital radius $6 \times 10^8 \text{ m}$, and orbital speed 3000 m/s, while the other has mass 84 kg, orbital radius $9 \times 10^8 \text{ m/s}$. What is the orbital speed of this second satellite?

EQUATIONS	CONSTANTS
$F_G = \frac{Gm_1m_2}{r^2} \quad r = R + h$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$g = \frac{GM}{r^2} \quad g_{\text{surf}} = \frac{GM}{R^2}$	$M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$
$v_{\text{sat}} = \sqrt{\frac{GM}{r}}$	

PRACTICE: You throw a baseball horizontally while on the surface of a small, spherical asteroid of mass $7 \times 10^{16} \text{ kg}$ and diameter of 22km. What is the minimum speed so that it just barely goes around the asteroid without hitting anything?

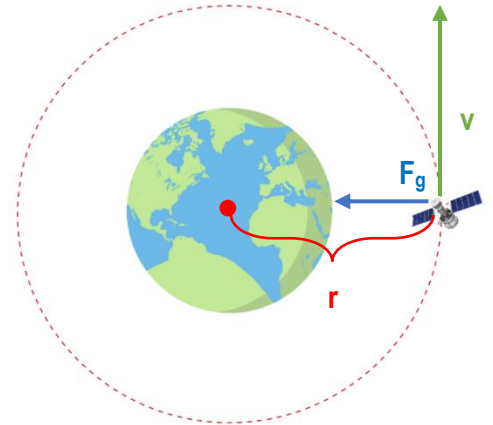
EQUATIONS	CONSTANTS
$F_G = \frac{Gm_1m_2}{r^2} \quad r = R + h$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$g = \frac{GM}{r^2} \quad g_{\text{surf}} = \frac{GM}{R^2}$	$M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$
$v_{\text{sat}} = \sqrt{\frac{GM}{r}}$	

CONCEPT: Orbital Period of a Satellite

- We can also relate the orbital speed and orbital period T , the time it takes to complete 1 orbit, using circular motion.

SATELLITE MOTION
$v_{\text{sat}} = \sqrt{\frac{GM}{r}}$

$v_{\text{sat}} = \underline{\hspace{2cm}}$
$T^2 = \underline{\hspace{2cm}}$



- This equation is also called vis-viva equation.
- Orbital speed, period, distance all interdependent. As distance increases, v decreases, T increases.

EXAMPLE: What is the orbital period and speed of the International Space Station orbiting 400km above Earth's surface?

SATELLITE MOTION	GRAV. CONSTANTS
$v_{\text{sat}} = \sqrt{\frac{GM}{r}} \quad v = \frac{2\pi r}{T}$ $T_{\text{sat}}^2 = \frac{4\pi^2 r^3}{GM}$	$G = 6.67 \times 10^{-11}$ $M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$

PRACTICE: A satellite orbits at an orbital period of 2 hours around the Moon. What is the satellite's orbital altitude?

EQUATIONS	CONSTANTS
$F_G = \frac{Gm_1m_2}{r^2}$ $r = R + h$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$g_{\text{near}} = \frac{GM}{R^2}$ $g_{\text{far}} = \frac{GM}{r^2}$	$M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$
$v_{\text{sat}} = \sqrt{\frac{GM}{r}}$ $v_{\text{sat}} = \frac{2\pi r}{T}$ $T_{\text{sat}}^2 = \frac{4\pi^2 r^3}{GM}$	$M_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg}$ $R_{\text{Moon}} = 1.74 \times 10^6 \text{ m}$

EXAMPLE: A small moon orbits its planet with a speed of 7500 m/s. It takes 28 hours to complete 1 full orbit. What is the mass of this unknown planet?

EQUATIONS	CONSTANTS
$F_G = \frac{Gm_1m_2}{r^2}$ $r = R + h$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$g_{\text{near}} = \frac{GM}{R^2}$ $g_{\text{far}} = \frac{GM}{r^2}$	$M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$
$v_{\text{sat}} = \sqrt{\frac{GM}{r}}$ $v_{\text{sat}} = \frac{2\pi r}{T}$ $T_{\text{sat}}^2 = \frac{4\pi^2 r^3}{GM}$	$M_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg}$ $R_{\text{Moon}} = 1.74 \times 10^6 \text{ m}$

PRACTICE: A distant planet orbits a star 3 times the mass of our Sun. This planet of mass $8 \times 10^{26} \text{ kg}$ feels a gravitational force of $2 \times 10^{26} \text{ N}$. What is this planet's orbital speed and how long does it take to orbit once?

EQUATIONS	CONSTANTS
$F_G = \frac{Gm_1m_2}{r^2}$ $r = R + h$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$g_{\text{near}} = \frac{GM}{R^2}$ $g_{\text{far}} = \frac{GM}{r^2}$	$M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$
$v_{\text{sat}} = \sqrt{\frac{GM}{r}}$ $v_{\text{sat}} = \frac{2\pi r}{T}$ $T_{\text{sat}}^2 = \frac{4\pi^2 r^3}{GM}$	$M_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg}$ $R_{\text{Moon}} = 1.74 \times 10^6 \text{ m}$ $M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$