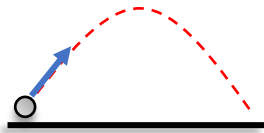


CONCEPT: SOLVING NON-SYMMETRICAL UPWARD LAUNCH PROBLEMS

- IF an object is launched upward and lands at a HIGHER or LOWER point, the motion is non-symmetrical.

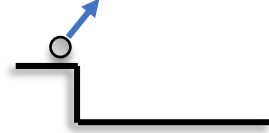
LANDS AT SAME HEIGHT
(Symmetrical Launch)



LANDS AT HIGHER HEIGHT
(Launch TO Height)

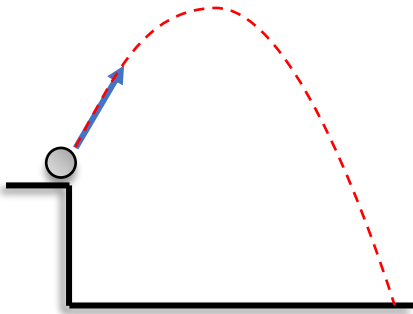


LANDS AT LOWER HEIGHT
(Launch FROM Height)



- IF landing at a LOWER height, part of motion (A→C) is symmetrical, but object drops further (C→D).
 - When choosing intervals in problems, **try** to include point ____ (max height) to simplify equations, because $v_{By} = \underline{\hspace{1cm}}$

EXAMPLE: You fire a potato from a launcher on a 20m-high cliff. The potato has an initial speed of 30m/s at 53° upwards. The potato reaches its max height 49.4m above the ground after 2.45s. Find the vertical component of the potato's velocity just before hitting the ground.



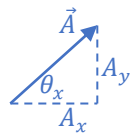
PROJECTILE MOTION

- 1) Draw paths in X&Y and points of interest
(Points of Interest: initial, final, max height, etc.)
- 2) Determine target variable
- 3) Determine interval and UAM equation
- 4) Solve

UAM EQUATIONS

X	Y
$\Delta x = v_x t$	(1) $v_y = v_{0y} + a_y t$ (2) $v_y^2 = v_{0y}^2 + 2a_y \Delta y$ (3) $\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$ *(4) $\Delta y = \frac{1}{2} (v_{0y} + v_f) t$

VECTOR EQs



$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta_x = \tan^{-1} \left(\frac{|A_y|}{|A_x|} \right)$$

$$A_x = A \cos(\theta_x)$$

$$A_y = A \sin(\theta_x)$$

PROBLEM: You throw a rock off the top of a tall building at an upward angle of 15° . At $t=3$ s, the rock's horizontal displacement from you is 52m. How high does the rock get above the top of the building?

- A) 1.1 m
- B) 4.6 m
- C) 30 m

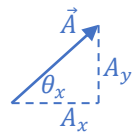
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$$A_x = A \cos(\theta_x)$$

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PROBLEM: A child throws a ball from ground level with an initial speed of 13 m/s at an upward angle of 67.4° . It reaches its maximum height directly above the edge of a roof, then lands on the roof, 3 m from the edge. How high is the roof?

- A) 7.3 m
- B) 2.9 m
- C) 5.6 m
- D) 4.4 m

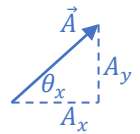
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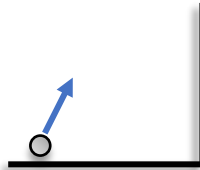
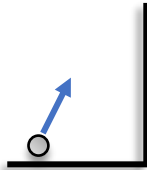
$$\theta_x = \tan^{-1} \left(\frac{|A_y|}{|A_x|} \right)$$

$$A_x = A \cos(\theta_x)$$

$$A_y = A \sin(\theta_x)$$

PROBLEM: A catapult launches a stone with an initial velocity of 50 m/s at an angle of 56° above the horizontal. What is the direction of the stone's velocity when it hits a castle wall 6 seconds later?

- A) -31.7°
- B) 31.7°
- C) -47.7°



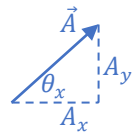
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VECTOR EQs



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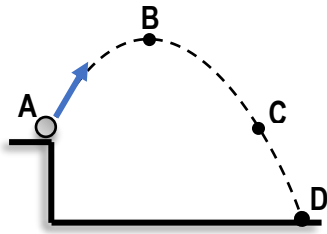
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CONCEPT: USING SINGLE INTERVALS IN UPWARD LAUNCH PROBLEMS

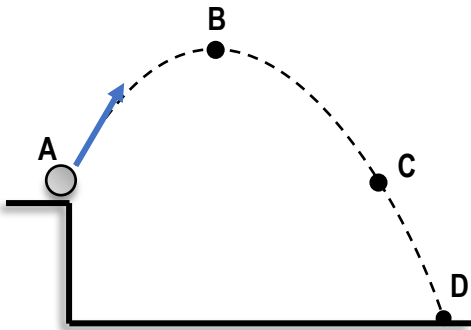
- In projectile motion, you can often choose different intervals (A→B, B→D, etc...) and still get the right answer!



$$\Delta t_{AD} = \underline{\hspace{2cm}} \text{ OR } \underline{\hspace{2cm}}$$

- Usually you should **try** to solve these problems using a single interval (___→___) because it's better/simpler/faster!

EXAMPLE: You fire a cannon with 100 m/s at 30° above the +x-axis from a 40-m cliff. Find **a)** the vertical component of the velocity at point when the cannonball hits the ground, and **b)** the total time of flight



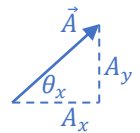
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PROBLEM: A ball is thrown from the top of a 50-m-tall building with a speed of 40m/s at an angle of 37° above the horizontal. How far horizontally does the ball travel before hitting the ground?

- A) 101.2 m
- B) 50.6 m
- C) 207 m
- D) 414 m

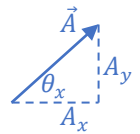
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PROBLEM: A ball is thrown from the top of a 50-m-tall building at an angle of 37° above the horizontal. 3 s later, it breaks a window at a lower height in a building 24 m away. How high above the ground is the window, to the nearest whole number?

- A) 26 m
- B) 52 m
- C) 24 m
- D) 62 m

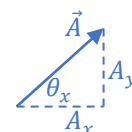
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