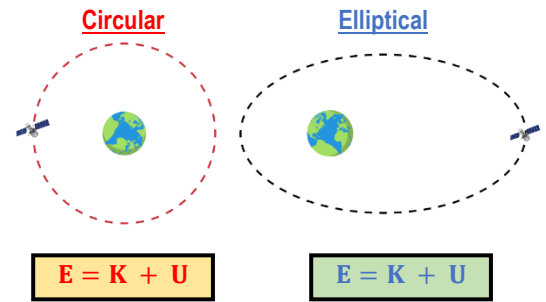


CONCEPT: Speeds & Energies in Elliptical Orbits

- Unlike for circular orbits, Kinetic & Potential energies are _____.
- Closest distance (PERI...), speed is [MAX | MIN] → K is [MAX | MIN]
- Because E_{ell} is conserved, when K is MAX, U is [MAX | MIN]
- Farthest distance (APO...), speed is [MAX | MIN] → K is [MAX | MIN]
- Because E_{ell} is conserved, when K is MIN, U is [MAX | MIN]



- To compare the speeds at two different points/distances in a fixed elliptical orbit:

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

- Comes from Conservation of _____ → $L =$

$$mv_1 r_1 = mv_2 r_2$$

EXAMPLE: A planet is observed orbiting a distant star. At its closest distance of 0.31 AU, its velocity is 59.0 km/s. What is its velocity at its farthest distance of 0.47AU from the star?

EQUATIONS	CONSTANTS
$a = \frac{R_a + R_p}{2}$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$R_a = a(1 + e)$	$1\text{AU} = 1.5 \times 10^{11} \text{ m}$
$R_p = a(1 - e)$	$M_E = 5.97 \times 10^{24} \text{ kg}$
$T_{\text{sat}}^2 = \frac{4\pi^2 a^3}{GM}$	$R_E = 6.37 \times 10^6 \text{ m}$
$U_G = -\frac{GMm}{r}$	$M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$
$K_i + U_i + W_{\text{NC}} = K_f + U_f$	$R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$
$E_{\text{circ}} = -\frac{GMm}{2r} = -\frac{1}{2}mv^2$	
$E_{\text{ell}} = -\frac{GMm}{2a} \quad v_a R_a = v_p R_p$	

- When plugging in for v and r , they can be non-SI if consistent (same units for v_1 & v_2 , r_1 & r_2)

PRACTICE: Pluto travels in a fairly elliptical orbit around the Sun. At its closest distance of 4.43×10^9 km, its orbital speed is 6.12 km/s. At its farthest, its orbital speed reduces to just 3.71 km/s. How far is it from the Sun at this point (in km)?

EQUATIONS	CONSTANTS
$a = \frac{R_a + R_p}{2}$ $R_a = a(1 + e)$ $R_p = a(1 - e)$ $T_{\text{sat}}^2 = \frac{4\pi^2 a^3}{GM}$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$ $M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$ $M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$ $R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$
$U_G = -\frac{GMm}{r}$ $K_i + U_i + W_{\text{NC}} = K_f + U_f$	
$E_{\text{circ}} = -\frac{GMm}{2r} = -\frac{1}{2}mv^2$ $E_{\text{ell}} = -\frac{GMm}{2a}$	
$v_1 r_1 = v_2 r_2$	

EXAMPLE: A comet travels around the Sun once every 2,000 years. At its closest approach, it is 3×10^8 km from the Sun. a) What is its farthest distance? b) What is the ratio of the comet's speed at the closest point to its farthest point?

EQUATIONS	CONSTANTS
$a = \frac{R_a + R_p}{2}$ $R_a = a(1 + e)$ $R_p = a(1 - e)$ $T_{\text{sat}}^2 = \frac{4\pi^2 a^3}{GM}$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$ $M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$ $M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$ $R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$
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$v_1 r_1 = v_2 r_2$	

CONCEPT: Energy Conservation in Changing Elliptical Orbits

- To go from a circular orbit $\leftrightarrow$ elliptical orbit, some work must be done.

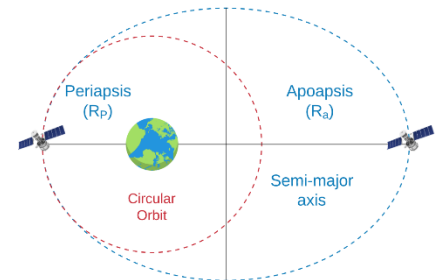
- Changing orbits, so we still use **Energy Conservation**:

$$K_i + U_i + W_{NC} = K_f + U_f \quad \rightarrow \quad \boxed{\text{_____} + W_{NC} = \text{_____}}$$

- Elliptical orbit energy equation is *very* similar to circular orbit, replace $r \rightarrow$ _____.

EXAMPLE: A spacecraft with mass 1×10^4 kg is in a 6.87×10^6 m circular orbit around the Earth. How much energy is required in order to reach an elliptical orbit with an apogee of 2.0×10^7 m?

Circular Orbits	Elliptical Orbits
$E_{\text{circ}} = -\frac{GMm}{2r}$	$E_{\text{ell}} = \text{_____}$



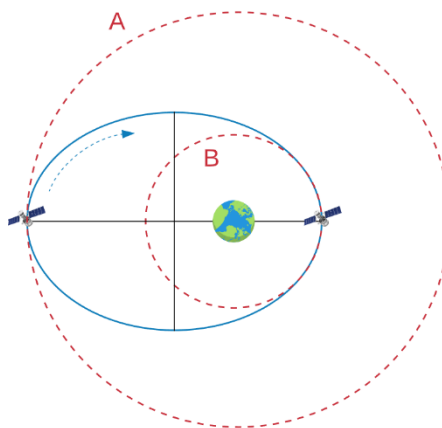
EQUATIONS	CONSTANTS
$a = \frac{R_a + R_p}{2}$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
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$E_{\text{circ}} = -\frac{GMm}{2r} = -\frac{1}{2}mv^2$	
$E_{\text{ell}} = -\frac{GMm}{2a}$	

- When changing from circular $\rightarrow$ elliptical orbits, r becomes _____, depending on the size of the new orbit.

- If new orbit is **larger**, $r =$ _____
- If new orbit is **smaller**, $r =$ _____

- From **elliptical** $\rightarrow$ **circular**:

- (A) If new orbit is **larger**, _____ = r
- (B) If new orbit is **smaller**, _____ = r



PRACTICE: A comet is in a highly elliptical orbit around the Sun with a period of 2400 years. If this comet has a mass of approximately 5×10^{14} kg, what is the total energy of its orbit?

EQUATIONS	CONSTANTS
$T_{\text{sat}}^2 = \frac{4\pi^2 a^3}{GM}$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
$U_G = -\frac{GMm}{r}$	$M_E = 5.97 \times 10^{24} \text{ kg}$
$K_i + U_i + W_{\text{NC}} = K_f + U_f$	$R_E = 6.37 \times 10^6 \text{ m}$
$E_{\text{circ}} = -\frac{GMm}{2r}$	$M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$
$E_{\text{ell}} = -\frac{GMm}{2a}$	$R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$

EXAMPLE: a) How much energy would be required for a 2,000-kg spacecraft to change from an orbit with a perigee of 6.87×10^6 m and apogee of 8.87×10^6 m to a circular orbit of 8.87×10^6 m? b) Assuming it fires its thrusters at apogee, by how much would it have to change its velocity to achieve this new orbit?

EQUATIONS	CONSTANTS
$a = \frac{R_a + R_p}{2}$ $R_a = a(1 + e)$ $R_p = a(1 - e)$ $T_{\text{sat}}^2 = \frac{4\pi^2 a^3}{GM}$	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$ $M_E = 5.97 \times 10^{24} \text{ kg}$ $R_E = 6.37 \times 10^6 \text{ m}$ $M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$ $R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$
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