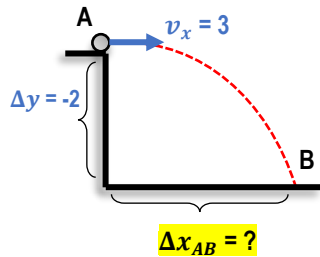


## CONCEPT: SOLVING PROJECTILE MOTION USING EQUATION SUBSTITUTION

**PREVIOUS EXAMPLE:** A ball rolls off a 2m-tall table at a speed of 3m/s. Find the horizontal displacement of the ball.



**X**

$$\Delta x_{AB} = v_x t_{AB}$$

$$\Delta x_{AB} = (3)(\quad)$$

**Y**

$$a_y = -g = -9.8$$

$$v_{0y} = v_{Ay} = 0$$

$$v_y = v_{By} =$$

$$\Delta y_{AB} = -2$$

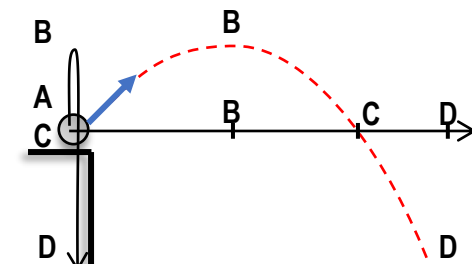
$$t_{AB} =$$

(3):  $\Delta y_{AB} = v_{Ay} t_{AB} + \frac{1}{2} a_y t_{AB}^2$

$t_{AB} = 0.64s$

- You'll need to solve problems where 2 of these 3 [ $v_0$ ,  $\theta_0$ ,  $t$ ] are unknown, but may not be asked for.
  - When you write X & Y equations, you may end up with        equations with the SAME        unknown variables.
  - Use **Substitution**: (1)        expression for 1 variable in *simplest* equation. Never write expression for  $\theta_A$  first!  
 (2)        expression in *other* equation.  
 (3) Solve for other unknown using *either* 1<sup>st</sup> or 2<sup>nd</sup> equation if necessary

**EXAMPLE:** A soccer ball is kicked upwards from a 5m-tall hill. It travels through the air for 4.5s and lands on the ground 45m away. Find the ball's initial velocity and the angle at which the soccer ball was kicked.



**X**

$$\Delta x = v_x t$$

**Y**

$$a_y = -g = -9.8$$

$$v_{0y} =$$

$$v_y =$$

$$\Delta y =$$

$$t =$$

### PROJECTILE MOTION

- Draw paths in X&Y and points of interest  
(Points of Interest: initial, final, max height, etc.)
- Determine target variable
- Determine interval and UAM equation
- Solve

#### UAM EQUATIONS

##### X

$$\Delta x = v_x t$$

##### Y

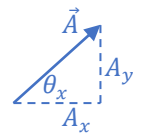
$$(1) v_y = v_{0y} + a_y t$$

$$(2) v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

$$(3) \Delta y = v_{0y} t + \frac{1}{2} a_y t^2$$

$$*(4) \Delta y = \frac{1}{2} (v_{0y} + v_f) t$$

#### VECTOR EQs



$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta_x = \tan^{-1} \left( \frac{|A_y|}{|A_x|} \right)$$

$$A_x = A \cos(\theta_x)$$

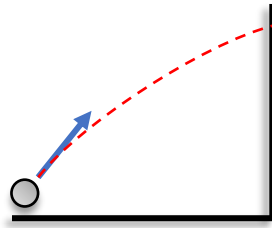
$$A_y = A \sin(\theta_x)$$

### EQUATION SUBSTITUTION

- Write expression for 1 variable in 1 equation
- Plug expression into other equation to solve
- Solve *other* variable in *either* equation

**PROBLEM:** A ball is kicked at a  $45^\circ$  angle from the ground. It hits the wall of a building 30m away, 10m up from the ground. What was the ball's initial velocity?

- A) 17.7 m/s
- B) 3.2 m/s
- C) 21 m/s
- D) 441 m/s



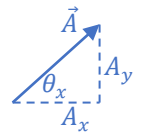
### PROJECTILE MOTION

- 1) Draw paths in X&Y and points of interest  
(Points of Interest: initial, final, max height, etc.)
- 2) Determine target variable
- 3) Determine interval and UAM equation
- 4) Solve

### UAM EQUATIONS

| X                  | Y                                                                                                                                                                         |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\Delta x = v_x t$ | $(1) v_y = v_{0y} + a_y t$<br>$(2) v_y^2 = v_{0y}^2 + 2a_y \Delta y$<br>$(3) \Delta y = v_{0y} t + \frac{1}{2} a_y t^2$<br>$*(4) \Delta y = \frac{1}{2} (v_{0y} + v_f) t$ |

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