

## CONCEPT: DIMENSIONAL ANALYSIS

- Equations work only if they are dimensionally consistent, meaning the units on both sides are \_\_\_\_\_.
- Easy way to check if equations make sense **without** calculations.

EXAMPLE: You walk a constant speed  $v = 5\text{m/s}$  for a time  $t$  of 2s. Which equation from below would be appropriate for determining the distance  $d$  in meters?

$$\begin{array}{l} \text{Distance} = \text{speed} \times \text{time} \\ d = v \times t \end{array}$$

OR

$$\begin{array}{l} \text{Distance} = \text{speed} \times \text{time}^2 \\ d = v \times 2t^2 \end{array}$$

$$[ \quad ] = [ \quad ] \times [ \quad ]$$

$$[ \quad ] = [ \quad ] \times [ \quad ]$$

### DIMENSIONAL CONSISTENCY

- 1) Replace variables with units
- 2) Ignore – signs & numbers (2,  $\frac{1}{2}$ , etc..)
- 3) Multiply & divide to cancel out units
- 4) Check if units on left = units on right

[ **CONSISTENT** | **INCONSISTENT** ]

[ **CONSISTENT** | **INCONSISTENT** ]

## DETERMINING UNITS OF UNKNOWN VARIABLES

- You'll also need Dimensional Analysis to figure out the units of unknown variables.

EXAMPLE: Hooke's Law states that a restoring Force  $F$ , measured in Newtons [N], in springs is related to the distance from equilibrium  $x$  by the equation  $F = -kx$ . What are the units of the force constant  $k$ ?

### SOLVING UNITS OF VARIABLES

- 1) Replace variables with units
- 2) Ignore – signs & numbers (2,  $\frac{1}{2}$ , etc..)
- 3) Isolate unknown variable
- 4) Solve

PRACTICE: A box moving with an initial speed  $v$  is accelerated horizontally. If  $x$  is measured in [m],  $v$  in [m/s],  $a$  in [m/s<sup>2</sup>],  $t$  in [s] which of the following equations is correct for solving the distance  $x$ ?

- A)  $x = \frac{a}{t^2}$
- B)  $x = v + \frac{1}{2} at$
- C)  $x = vt + \frac{1}{2} at^2$

PRACTICE: Newton's Law of Gravitation describes the attraction force between two masses. The equation is

$F = G \frac{m_1 m_2}{r^2}$ , where  $F$  is in [ $\frac{\text{kg} \cdot \text{m}}{\text{s}^2}$ ],  $m_1$  and  $m_2$  are masses in [kg], and  $r$  is the distance in [m] between them.

Determine the units of the Universal Constant  $G$ .

- A)  $\frac{\text{kg} \cdot \text{s}^2}{\text{m}^3}$
- B)  $\frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
- C)  $\frac{\text{m}}{\text{s}^2}$
- D)  $\frac{\text{m}^3}{\text{s}^2}$