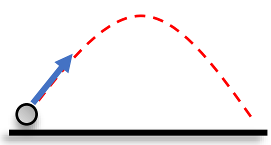
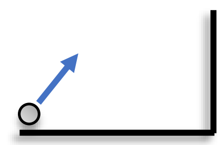
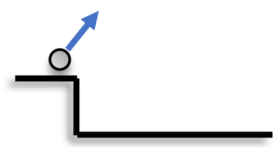
CONCEPT: SOLVING NON-SYMMETRICAL UPWARD LAUNCH PROBLEMS

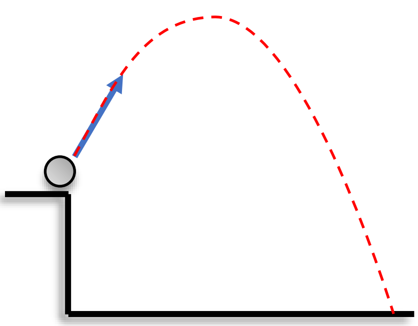
● **IF** an object is launched upward and lands at a HIGHER **or** LOWER point, the motion is **non-symmetrical.**

LANDS AT SAME HEIGHT (Symmetrical Launch)	**LANDS AT HIGHER HEIGHT** (Launch *TO* Height)	**LANDS AT LOWER HEIGHT** (Launch *FROM* Height)
		

● **IF** landing at a LOWER height, part of motion (A⟶C) is symmetrical, but object drops further (C⟶D).

- When choosing intervals in problems, ***try*** to include point ___ (max height) to simplify equations, because v_{By} = ___

EXAMPLE: You fire a potato from a launcher on a 20m-high cliff. The potato has an initial speed of 30m/s at 53° upwards. The potato reaches its max height 49.4m above the ground after 2.45s. Find the vertical component of the potato's velocity just before hitting the ground.

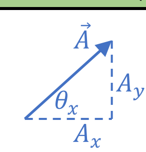


PROJECTILE MOTION

1) Draw paths in X&Y and points of interest
(Points of Interest: initial, final, max height, etc.)
2) Determine target variable
3) Determine interval and UAM equation
4) Solve

UAM EQUATIONS	
X	**Y**
$\Delta x = v_x t$	(1) $v_y = v_{0y} + a_y t$ (2) $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ (3) $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ *(4) $\Delta y = \frac{1}{2}(v_{0y} + v_f)t$

VECTOR EQs



$A = \sqrt{A_x^2 + A_y^2}$

$\theta_x = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right)$

$A_x = A\cos(\theta_x)$

$A_y = A\sin(\theta_x)$

PROBLEM: You throw a rock off the top of a tall building at an upward angle of 15°. At t=3 s, the rock's horizontal displacement from you is 52m. How high does the rock get above the top of the building?

A) 1.1 m
B) 4.6 m
C) 30 m

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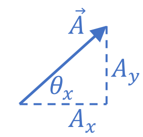

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<u>PROBLEM</u>**:** A child throws a ball from ground level with an initial speed of 13 m/s at an upward angle of 67.4°**.** It reaches its maximum height directly above the edge of a roof, then lands on the roof, 3 m from the edge**.** How high is the roof?

- **A)** 7.3 m
- **B)** 2.9 m
- **C)** 5.6 m
- **D)** 4.4 m

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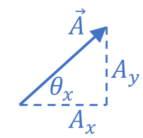

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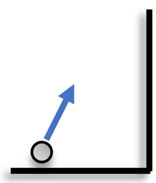
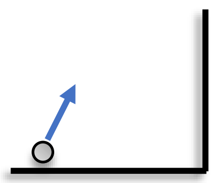
PROBLEM: A catapult launches a stone with an initial velocity of 50 m/s at an angle of 56° above the horizontal. What is the direction of the stone's velocity when it hits a castle wall 6 seconds later?

A) –31.7°
B) 31.7°
C) –47.7°





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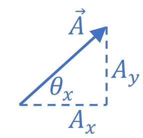

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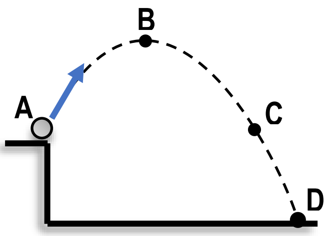
CONCEPT: USING SINGLE INTERVALS IN UPWARD LAUNCH PROBLEMS

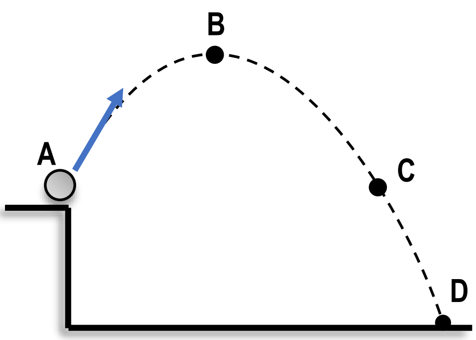
● In projectile motion, you can often choose different intervals (A⟶B, B⟶D, etc…) and still get the right answer!



$\Delta t_{AD} =$ ___________ OR ____________

- Usually you should ***try*** to solve these problems using a *single* interval (___⟶___) because it's better/simpler/faster!

EXAMPLE**:** You fire a cannon with 100 m/s at 30° above the +x-axis from a 40-m cliff**.** Find **a)** the vertical component of the velocity at point when the cannonball hits the ground, and **b)** the total time of flight



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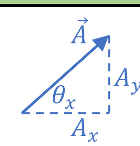

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PROBLEM: A ball is thrown from the top of a 50-m-tall building with a speed of 40m/s at an angle of 37° above the horizontal. How far horizontally does the ball travel before hitting the ground?

A) 101.2 m
B) 50.6 m
C) 207 m
D) 414 m

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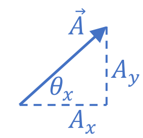

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PROBLEM: A ball is thrown from the top of a 50-m-tall building at an angle of 37° above the horizontal. 3 s later, it breaks a window at a lower height in a building 24 m away. How high above the ground is the window, to the nearest whole number?

A) 26 m
B) 52 m
C) 24 m
D) 62 m

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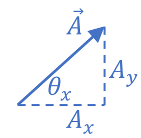

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