CONCEPT: FINDING MOMENT OF INERTIA BY INTEGRATING

● The moment of inertia of an object can be found by knowing the formula for a particular situation
- However, what if you don't have a formula for a particular situation? Plus, where do those formulas come from?
- You can find the moment of inertia for any object about any axis by using integration

● For a point mass, dm, at a distance r from the rotation axis, the moment of inertia is $dI = r^2 dm$
- A solid object is made up of an infinite number of these infinitesimal masses, dm, each at a different r
- To find the total moment of inertia, we need to sum all of the dI's, or integrate

● The MOMENT OF INERTIA of some object, about some axis, is

$I =$ ________________ where r will TYPICALLY change with m

EXAMPLE 1: What is the moment of inertia for a ring of mass m and radius R, rotating about an axis through its center, perpendicular to the surface of the ring? The mass is uniformly distributed throughout the ring.

● This integral isn't USUALLY as simple as pulling r^2 out of the integral and saying $\int dm = m$
- Since mass is distributed across all radii, we need to find a way to relate dm to dr

EXAMPLE 2: What is the moment of inertia of a disk of mass m and radius R, rotating about an axis through its center, perpendicular to the surface of the ring? The mass is uniformly distributed throughout the disk.

EXAMPLE: MOMENT OF INERTIA OF A NON-UNIFORM DISK

What is the moment of inertia of a NON-UNIFORM disk, of mass m and radius R, about an axis through its center, perpendicular to the surface of the disk. The mass distribution is given by $\sigma = \alpha r^2$. Give your answer entirely in terms of m and R.