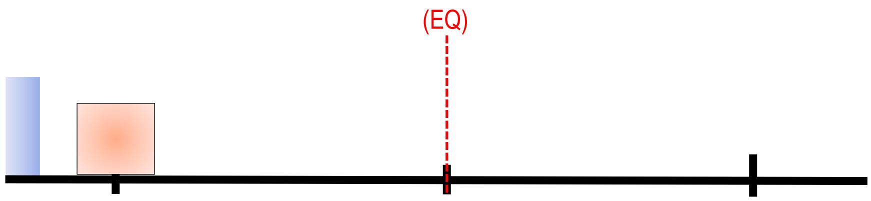
CONCEPT: Intro to Simple Harmonic Motion

● The most common type of Simple Harmonic Motion (aka Oscillation) is the mass-spring system.



x = ______ = ______	**x** = ______	**x** = ______ = ______
v = ______	**v** = ______	**v** = ______
F = ______	**F** = ______	**F** = ______
a = ______	**a** = ______	**a** = ______

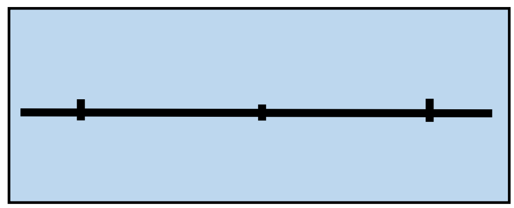
● Amplitude → A:
- ____________ displacement, **|x|**
- always the _________________.

● Period → T [seconds/cycle]
- Time for one ____________ cycle.

● Frequency → f = 1/T [cycles/second]

● Angular frequency → ω [rad/second]
- ω = _______ = _______

<u>EXAMPLE 1</u>**:** A mass on a spring is pulled 1m away from its equilibrium position, then released from rest**.** The mass takes 2s to reach maximum displacement on the other side**.** Calculate the **(a)** amplitude, **(b)** period, **(c)** angular frequency of the motion.



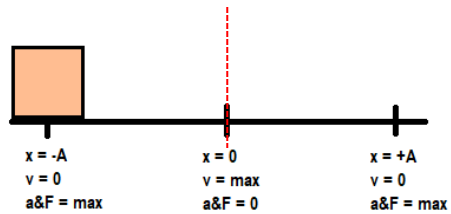
<u>PRACTICE:</u> A mass-spring system with an angular frequency ω = 8π rad/s oscillates back and forth**.** (**a**) Assuming it starts from rest, how much time passes before the mass has a speed of 0 again? (**b**) How many full cycles does the system complete in 60s?

Mass-Spring SHM Equations
$\lvert F_S \rvert = \lvert F_A \rvert = kx$ $a = -\frac{k}{m}x$
$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$ $N\ [\text{cycles}] = \frac{t\ [\text{time}]}{T\ [\text{Period}]} = t * f$

CONCEPT: Equations for Simple Harmonic Motion

● In Simple Harmonic Motion, acceleration NOT constant → kinematic equations?

Old Equations → x (position)	
$F_s = -k\,x$	→ F_{max} = _____
$a = -\frac{k\,x}{m}$	→ a_{max} = _____
New Equations → t (time)	
$x(t) = +\ A\cos(\omega t)$	→ x_{max} = _____
$v(t) = -\ A\omega\ \sin(\omega t)$	→ v_{max} = _____
$a(t) = -A\omega^2\cos(\omega t)$	→ a_{max} = _____

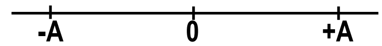


- Calculator must be in _________.

● Combining $a_{max}(x)$ and $a_{max}(t)$ → $\omega = 2\pi f = \frac{2\pi}{T} =$ _______

EXAMPLE 1: A 4-kg mass is attached to a spring where k = 200[N/m]. The mass is pulled 2m and released from rest. Find the (**a**) angular frequency, (**b**) velocity 0.5s after release, (**c**) acceleration when x = 0.5m, and (**d**) the period of oscillation.

PRACTICE: A 4-kg mass on a spring is released 5 m away from equilibrium position and takes 1.5 s to reach its equilibrium position. (**a**) Find the spring's force constant. (**b**) Find the object's max speed.



Mass-Spring SHM Equations

$|F_S| = |F_A| = kx$ → $F_{max} = \pm kA$

$a = -\frac{k}{m}x$ → $a_{max} = \pm\frac{k}{m}A$

$x(t) = +\ A\cos(\omega t)$ → $x_{max} = \pm A$

$v(t) = -A\omega\ \sin(\omega t)$ → $v_{max} = \pm A\omega$

$a(t) = -A\omega^2\cos(\omega t)$ → $a_{max} = \pm A\omega^2$

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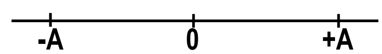
$$N\ [\text{cycles}] = \frac{t\ [\text{time}]}{T\ [\text{Period}]} = t * f$$

EXAMPLE: A 4-kg mass is attached to a horizontal spring and oscillates at 2 Hz. If mass is moving with 10 m/s when it crosses its equilibrium position, (**a**) how long does it take to get from equilibrium to its max distance? Find the (**b**) amplitude and (**c**) maximum acceleration.



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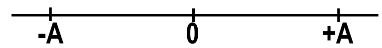
PRACTICE: What is the equation for the position of a mass moving on the end of a spring which is stretched 8.8cm from equilibrium and then released from rest, and whose period is 0.66s? What will be the object's position after 1.4s?



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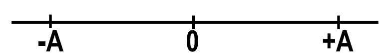
EXAMPLE: The velocity of a particle on a spring is given by the equation v(t) = -6.00 sin (3π t). Determine the (**a**) frequency of motion, (**b**) amplitude, and (**c**) velocity of the particle at t = 0.5s.



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