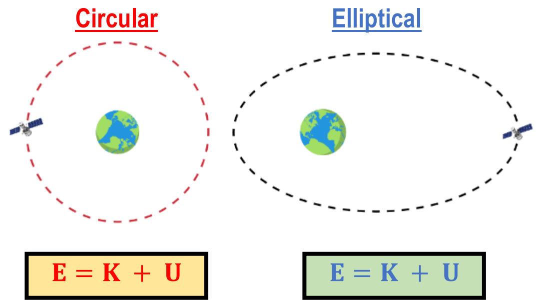
CONCEPT: Speeds & Energies in Elliptical Orbits



● Unlike for circular orbits, Kinetic & Potential energies are ____________.

- Closest distance (PERI…), speed is **[MAX | MIN]** → **K** is **[MAX | MIN]**
- Because $\mathbf{E_{ell}}$ is conserved, when K is **MAX**, U is **[MAX | MIN]**

- Farthest distance (APO…), speed is **[MAX | MIN]** → **K** is **[MAX | MIN]**
- Because $\mathbf{E_{ell}}$ is conserved, when K is **MIN,** U is **[MAX | MIN]**

● To compare the speeds at two different points/distances in a fixed elliptical orbit: **______ = ______**

- Comes from Conservation of __________________________ $\rightarrow \boldsymbol{L} =$

$$\mathbf{mv_1r_1 = mv_2r_2}$$

EXAMPLE: A planet is observed orbiting a distant star. At its closest distance of 0.31 AU, its velocity is 59.0 km/s. What is its velocity at its farthest distance of 0.47AU from the star?

EQUATIONS	CONSTANTS
$a = \frac{R_a + R_p}{2}$ $R_a = a(1+e)$ $R_p = a(1-e)$ $T_{sat}^2 = \frac{4\pi^2 a^3}{GM}$	G = 6.67×10⁻¹¹ $\frac{m^3}{kg \cdot s^2}$ 1AU = 1.5×10¹¹ m M_E = 5.97×10²⁴ kg R_E = 6.37×10⁶ m M_{Sun} = 2×10³⁰ kg R_{Sun} = 6.96×10⁸ m
$U_G = -\frac{GMm}{r}$ $K_i + U_i + W_{NC} = K_f + U_f$	
$E_{circ} = -\frac{GMm}{2r} = -\frac{1}{2}mv^2$ $E_{ell} = -\frac{GMm}{2a}$ $v_a R_a = v_p R_p$	

- When plugging in for **v** and **r**, they can be non-SI if consistent (same units for $\mathbf{v_1}$ & $\mathbf{v_2}$, $\mathbf{r_1}$ & $\mathbf{r_2}$)

PRACTICE: Pluto travels in a fairly elliptical orbit around the Sun. At its closest distance of 4.43×10^9 km, its orbital speed is 6.12 km/s. At its farthest, its orbital speed reduces to just 3.71 km/s. How far is it from the Sun at this point (in km)?

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EXAMPLE: A comet travels around the Sun once every 2,000 years. At its closest approach, it is 3×10^8 km from the Sun. a) What is its farthest distance? b) What is the ratio of the comet's speed at the closest point to its farthest point?

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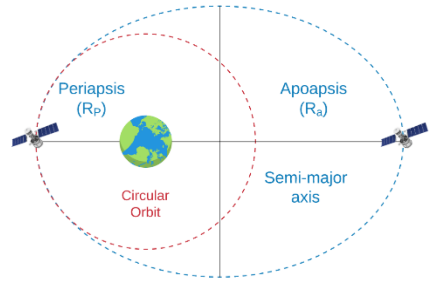
CONCEPT: Energy Conservation in Changing Elliptical Orbits

Circular Orbits	Elliptical Orbits
$E_{circ} = -\frac{GMm}{2r}$	$E_{ell} =$ ______

● To go from a circular orbit ↔ *elliptical* orbit, some work must be done.

- Changing orbits, so we still use **Energy Conservation:**

$$K_i + U_i + W_{NC} = K_f + U_f \quad \rightarrow \quad \_\_\_\_\_ + W_{NC} = \_\_\_\_\_$$



● Elliptical orbit energy equation is *very* similar to circular orbit, replace **r** → ____.

EXAMPLE: A spacecraft with mass 1×10^4 kg is in a 6.87×10^6 m circular orbit around the Earth. How much energy is required in order to reach an elliptical orbit with an apogee of 2.0×10^7 m?

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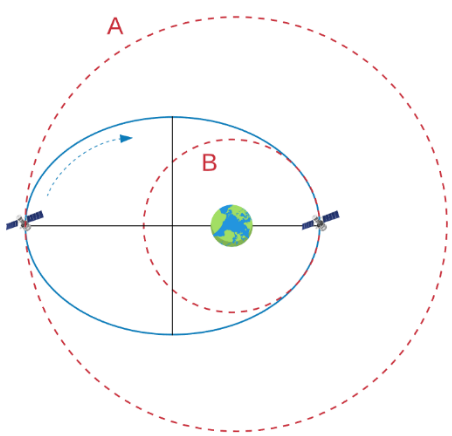
● When changing from circular → elliptical orbits, **r** *becomes* ______________, depending on the size of the new orbit.

- If new orbit is **larger**, r = _____
- If new orbit is **smaller**, r = _____

- From elliptical → circular:
  - **(A)** If new orbit is **larger**, ____ = r
  - **(B)** If new orbit is **smaller**, ____ = r



PRACTICE: A comet is in a highly elliptical orbit around the Sun with a period of 2400 years. If this comet has a mass of approximately 5×10^{14} kg, what is the total energy of its orbit?

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EXAMPLE: a) How much energy would be required for a 2,000-kg spacecraft to change from an orbit with a perigee of 6.87×10^6 m and apogee of 8.87×10^6 m to a circular orbit of 8.87×10^6 m? b) Assuming it fires its thrusters at apogee, by how much would it have to change its velocity to achieve this new orbit?

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