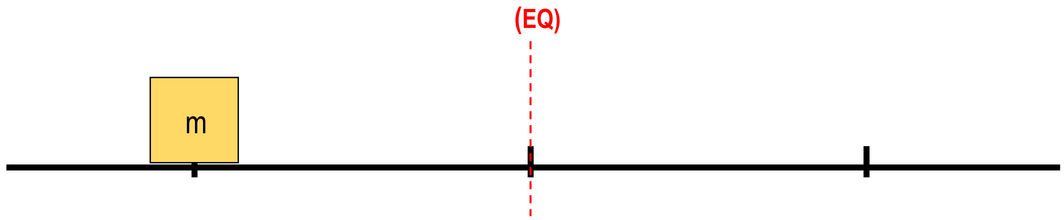
CONCEPT: Energy in Simple Harmonic Motion

● At any point of SHM, the mass-spring system may have 2 types of Energy: ___________ + ____________.

- W_{nc} = _____, so the Mechanical Energy **(M.E.)** is _______________.



● Energy Conservation → always compare energies at 2 special points:		**Any other Point:**
Amplitude:	**Equilibrium:**	
x = ______	x = ______	x = ______
Elastic Energy (U_A) = $\frac{1}{2}kx^2$ = ______	Elastic Energy (U_0) = $\frac{1}{2}kx^2$ = ______	Elastic Energy (U_P) = $\frac{1}{2}kx^2$ = ______
Kinetic Energy (K_A) = $\frac{1}{2}mv^2$ = ______	Kinetic Energy (K_0) = $\frac{1}{2}mv^2$ = ______	Kinetic Energy (K_P) = $\frac{1}{2}mv^2$ = ______
Total M.E. = ______________	**Total M.E. = ______________**	**Total M.E. = ______________**

● Comparing all these energies at different points:

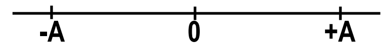
$U_A = K_0 = U_P + K_P$

_________ = __________ = __________ + __________	→ $v(x) =$ __________________

(Energy Conservation for Springs)

EXAMPLE 1**:** A 5 kg mass oscillates on a horizontal spring with k = 30[N/m] and an amplitude of 0.4 m**.** Find its (**a**) max speed, (**b**) speed when it is at -0.2 m, and (**c**) the total mechanical energy of the system**.**

EXAMPLE: A 0.25-kg mass oscillates on a spring with a period of 3.2s. At x=0.4m, it is observed to have a speed of 5m/s. What is the system's (**a**) Amplitude and (**b**) total mechanical energy?



Mass-Spring SHM Equations

$|F_S| = |F_A| = kx \quad \rightarrow F_{max} = \pm kA$

$a = -\frac{k}{m}x \quad \rightarrow a_{max} = \pm\frac{k}{m}A$

$x(t) = +\ A\cos(\omega t) \quad \rightarrow x_{max} = \pm A$

$v(t) = -A\omega\ \sin(\omega t) \quad \rightarrow v_{max} = \pm A\omega$

$a(t) = \ -A\omega^2\cos(\omega t) \quad \rightarrow a_{max} = \pm A\omega^2$

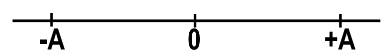
$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$N\ [\text{cycles}] = \frac{t\ [\text{time}]}{T\ [\text{Period}]} = t * f$$

$$M.E. = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kx_p^2 + \frac{1}{2}mv_p^2$$

$$v(x) = \omega\sqrt{A^2 - x^2}$$

PRACTICE: A block of mass 0.300 kg is attached to a spring. At x = 0.240 m, its acceleration is a_x = -12.0 m/s^2 and its velocity is v_x = 4.00 m/s. What are the system's (**a**) force constant k and (**b**) amplitude of motion?



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<u>EXAMPLE</u>**:** You increase the amplitude of oscillation of a mass vibrating on a spring**.** Which statements are correct?

(a) Period of oscillation increases

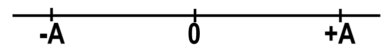
(b) Maximum acceleration increases

(c) Maximum speed increases

(c) Max Kinetic Energy increases

(d) Max Potential Energy increases

(e) Max Total Energy increases



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