TOPIC: SUM AND DIFFERENCE IDENTITIES

Sum and Difference of Sine & Cosine

◆ The sum & difference identities are useful when you have multiple angles in the argument of a trig function.

TRIG IDENTITIES			
Name	**Identity**	**Example**	**Use when...**
Sum & Difference	$\sin(a+b) = \_\_\_\_\_\boldsymbol{a}\ \_\_\_\_\_\boldsymbol{b} \qquad \_\_\_\_\_\boldsymbol{a}\ \_\_\_\_\_\boldsymbol{b}$	$\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$ OR $\sin(90° + 30°)$	argument contains a _____/_____ *OR* multiples of $15°$ or $\frac{\pi}{12}$
	$\sin(a-b) = \sin\boldsymbol{a}\ \cos\boldsymbol{b} \qquad \cos\boldsymbol{a}\ \sin\boldsymbol{b}$		
	$\cos(a+b) = \_\_\_\_\_\boldsymbol{a}\ \_\_\_\_\boldsymbol{b} \qquad \_\_\_\_\_\boldsymbol{a}\ \_\_\_\_\_\boldsymbol{b}$		
	$\cos(a-b) = \cos\boldsymbol{a}\ \cos\boldsymbol{b} \qquad \sin\boldsymbol{a}\ \sin\boldsymbol{b}$		

◆ To find exact values of trig functions NOT on the unit circle, rewrite argument as sum or diff. of 2 KNOWN angles.

EXAMPLE Find the exact value of the function.

$\cos 15°$

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EXAMPLE Rewrite each argument as the sum or difference of two angles on the unit circle.

$(\boldsymbol{A})$ $\sin(75°)$

$(\boldsymbol{B})$ $\cos(-15°)$

$(\boldsymbol{C})$ $\cos\left(\frac{7\pi}{12}\right)$

PRACTICE Find the exact value of the expression.

$(\boldsymbol{A})$ $\cos 105°$

Recall

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

(*Sum/Difference Identities*)

$(\boldsymbol{B})$ $\sin 15°$

$(\boldsymbol{C})$ $\cos\frac{5\pi}{12}$

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EXAMPLE Find the exact value of the expression.

$\sin 10° \cos 20° + \sin 20° \cos 10°$

Recall

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

(*Sum/Difference Identities*)

PRACTICE Find the exact value of the expression.

$\cos 80° \cos 20° + \sin 80° \sin 20°$

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EXAMPLE Expand the expression using the sum & difference identities and simplify.

(A)

$\sin(\theta + 30°)$

Recall

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

(Sum/Difference Identities)

(B)

$\cos\left(\frac{\pi}{4} - \theta\right)$

PRACTICE Expand the expression using the sum & difference identities and simplify.

$\sin\left(-\theta - \frac{\pi}{2}\right)$

TOPIC: SUM AND DIFFERENCE IDENTITIES

Sum and Difference of Tangent

◆ The sum & diff. identities for tangent can also be used to simplify expressions & find exact values of functions.

TRIG IDENTITIES

Name	Identity	Example	Use when...
Sum & Diff.	$\tan(a+b) = \dfrac{\tan a \quad \tan b}{1 \quad \tan a \tan b}$	$\tan\left(\pi + \frac{\pi}{4}\right)$	argument contains a $+$ / $-$ *OR* multiples of $15°$ or $\frac{\pi}{12}$
	$\tan(a-b) = \dfrac{\tan a \quad \tan b}{1 \quad \tan a \tan b}$		

EXAMPLE Expand the expression using the sum and/or difference identities, then simplify.

(A) $\tan(\theta - 45°)$

(B) $\tan(90° + \theta)$

◆ If $\tan a$ or $\tan b$ is undefined, rewrite $\tan(a \pm b)$ as $\dfrac{\sin(a \pm b)}{\cos(a \pm b)}$

TOPIC: SUM AND DIFFERENCE IDENTITIES

PRACTICE Find the exact value of the expression.

$\tan 105°$

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(*Sum/Difference Identities*)

PRACTICE Expand the expression using the sum & difference identities and simplify.

$\tan\left(-\theta - \frac{\pi}{2}\right)$

TOPIC: SUM AND DIFFERENCE IDENTITIES

Verifying Identities Using Sum and Difference Formulas

◆ Recall: To verify an identity, simplify one or both sides to make them equal. Start with more complicated side.

EXAMPLE Verify the identity using the sum/diff. identities.

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

STRATEGIES: Simplifying Trig Expressions

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(*Sum/Difference Identities*)

EXAMPLE Verify the identity using the sum/difference identities.

$$\cot(90° - \theta) = \tan\theta$$

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EXAMPLE Verify the identity.

(A)

$$\frac{\cos(a+b)}{\cos a \cos b} = -\tan a \tan b + 1$$

STRATEGIES: Simplifying Trig Expressions

- Constantly scan for identities
- Add fractions using a common denominator
- Break down in terms of sin & cos
- If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- Factor

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(*Sum/Difference Identities*)

(B)

$$\frac{\sin(a-b)}{\tan a \tan b} = \cos a \cos b\,(\cot b - \cot a)$$

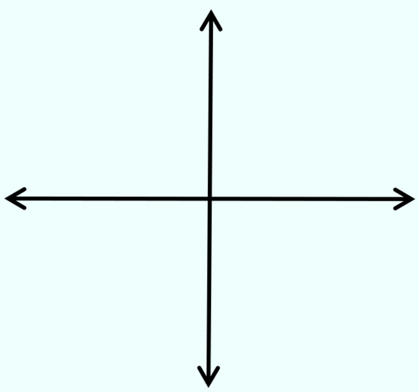
TOPIC: SUM AND DIFFERENCE IDENTITIES

Evaluate Sums and Differences Given Conditions

◆ Sometimes you'll need to evaluate a sum or difference based on given trig values without knowing the angles.

EXAMPLE

Find $\sin(a+b)$ given $\cos a = \frac{4}{5}, \sin b = \frac{5}{13}$, where a is in Quadrant IV & b is in Quadrant II.



HOW TO: Evaluate Trig Functions Given Conditions

1) _________ identity & **identify** unknown trig value(s)

2) From given info, sketch & label* ⊿(s) in *proper quadrant*

3) Find ______________(s) using Pythag. Thm.*

4) Solve for unknown trig value(s) from **(1)**

5) Plug in values & simplify

**pay attention to* _______

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(*Sum/Difference Identities*)

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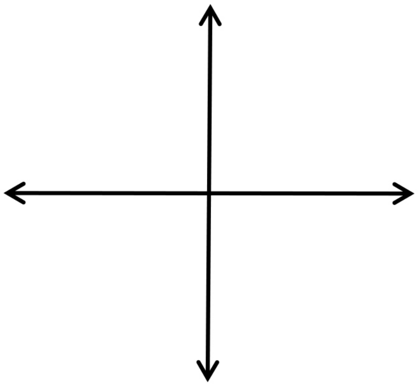
EXAMPLE Find $\sin(a+b)$ given $\sin a = \frac{3}{5}, \sin b = -\frac{1}{2}$, where $\frac{\pi}{2} < a < \pi$ & $\frac{3\pi}{2} < b < 2\pi$



HOW TO: Evaluate Trig Functions Given Conditions

1) Expand identity & **identify** unknown trig value(s)

2) From given info, sketch & label* ⊿(s) in *proper quadrant*

3) Find missing sides using Pythag. Thm.*

4) Solve for unknown trig value(s) from **(1)**

5) Plug in values & simplify

**pay attention to SIGN*

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(*Sum/Difference Identities*)

PRACTICE Find $\cos(a+b)$ given $\cos a = \frac{1}{2}$, $\sin b = \frac{1}{2}$, & a is in QIV & b is in QII.