TOPIC: GRAPHING RATIONAL FUNCTIONS

Graphing Rational Functions Using Transformations

- We can graph many rational functions by applying rules of transformations to $\frac{1}{x}$

Reflect over [$[x \mid y]$ axis, $[x \mid y]$ axis] **shift** [[Horiz. \| Vert.], [Horiz. \| Vert.]]

$$g(x) = -f(-x - h) + k$$

(*Transformations*)

$$g(x) = \frac{1}{x-3} + 1$$

TO GRAPH

1) Plot VA at $x =$ ____: $x =$ _____

2) Plot HA at $y =$ ____: $y =$ _____

3) a. Reflect? □ → (1,1) (−1,−1) over $[x \mid y]$

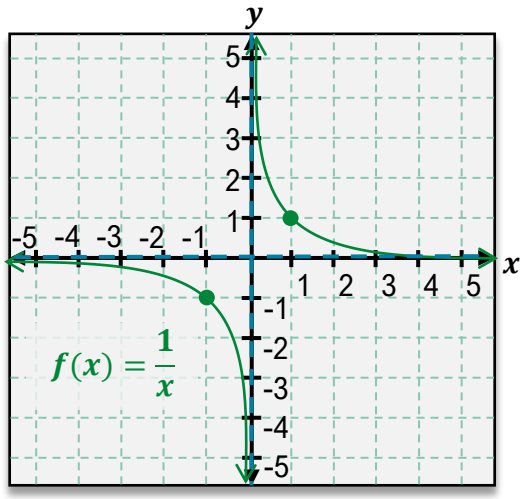
b. Shift test points by (____, ____)

4) Sketch curves approaching asymptotes

FROM GRAPH

Domain: (−∞ , ____) ∪ (____, ∞)

Range: (−∞ , ____) ∪ (____, ∞)



EXAMPLE: Graph the given function as a transformation of $f(x) = \frac{1}{x^2}$

$$g(x) = -\frac{1}{(x-2)^2}$$

TO GRAPH

1) Plot VA at $x = h$: $x =$ _____

2) Plot HA at $y = k$: $y =$ _____

3) a. Reflect? □ → (1,1) (−1,1) over $[x \mid y]$

b. Shift test points by (____, ____)

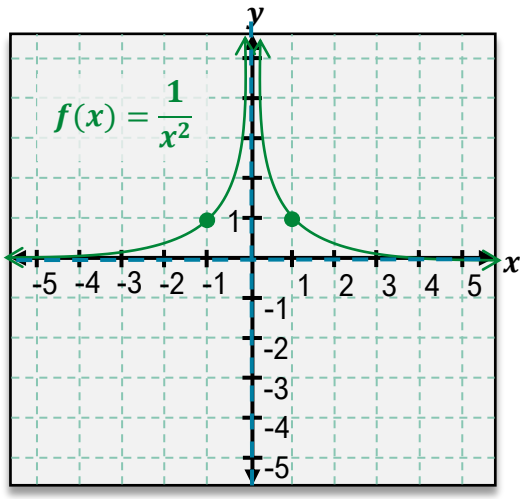
4) Sketch curves approaching asymptotes

FROM GRAPH

Domain: (−∞ , ____) ∪ (____ , ∞)
h h

Range: If upper quadrants: (k, ∞)

If lower quadrants: (−∞ , ____)
k



TOPIC: GRAPHING RATIONAL FUNCTIONS

PRACTICE: Graph the rational functions using transformations.

(A)

$$g(x) = -\frac{1}{x} + 3$$

TO GRAPH

1) Plot VA at $x = h$: $x =$ _____

2) Plot HA at $y = k$: $y =$ _____

3) a. Reflect? □ → $(1,1)$ $(-1,-1)$ over $[x \mid y]$

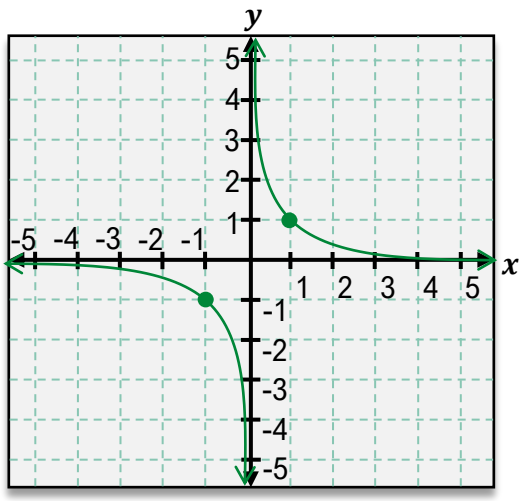
b. Shift test points by (____, ____)

4) Sketch curves approaching asymptotes

FROM GRAPH

Domain: (____ , ____) ∪ (____, ____)

Range: (____ , ____) ∪ (____, ____)



PRACTICE: Graph the rational functions using transformations.

(B)

$$g(x) = \frac{1}{(x+3)^2} - 2$$

TO GRAPH

1) Plot VA at $x = h$: $x =$ _____

2) Plot HA at $y = k$: $y =$ _____

3) a. Reflect? □ → $(1,1)$ $(-1,1)$ over $[x \mid y]$

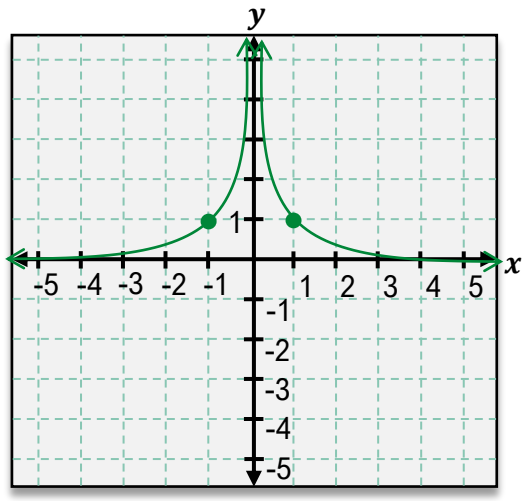
b. Shift test points by (____, ____)

4) Sketch curves approaching asymptotes

FROM GRAPH

Domain: (____ , ____) ∪ (____, ____)

Range: (____ , ____)



TOPIC: GRAPHING RATIONAL FUNCTIONS

How to Graph Rational Functions

- To graph a rational function, determine **vertical & horizontal asymptotes**, **x** & **y-intercepts**, and **holes**.

TO GRAPH

$$f(x) = \frac{2x-3}{x-1}$$

1) Factor & find Domain → Set Denom. = 0

$x - 1 = 0$ $\quad \{x | x \neq$ _____$\}$

2) Holes → a. Set Common Factor(s) = 0:____

b. Put in Lowest Terms

3) x-int(s) & behavior→ Set Num. $= 0$ (to solve $f(x) = 0$)

$2x - 3 = 0$ $\quad x =$ _____

Multiplicity: ____

EVEN ODD

[TOUCH | CROSS]

4) y-int → Compute $f(0)$: _____

$f(0) = \frac{2(0)-3}{0-1} =$

5) Vertical Asymptote(s) → Set Denom. = 0

$x - 1 = 0$ $\quad x =$ _____

6) Horizontal/Slant Asymptote(s) $\frac{2x-3}{x-1}$

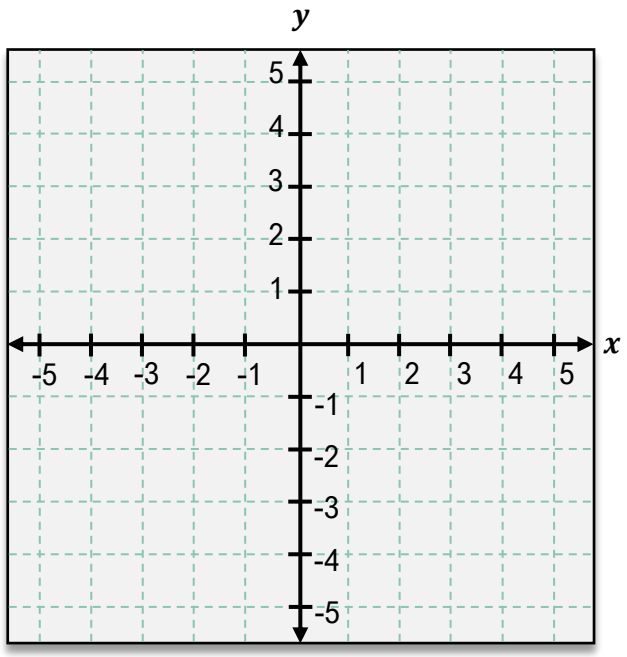
Deg. Num. [< | =] Deg. Denom

$y =$ [0 | divide lead coeff] → _____

7) Determine intervals & plot a point in each

x			
$f(x)$			

8) Connect & draw approaching asymptotes



- Recall: Find behavior *between* known components by breaking graph into _____.
  - Use vertical asymptote(s) & x-intercept(s) to form intervals.

TOPIC: GRAPHING RATIONAL FUNCTIONS

EXAMPLE: Graph the rational function.

$$f(x) = \frac{2x^2}{x^2 - 1}$$

TO GRAPH

1) Factor & find Domain → Set Denom. = 0

$\{x | x \neq$ _____$\}$

2) Holes → a. Set Common Factor(s) = 0:____

b. Put in Lowest Terms

3) x-int(s) & behavior→ Set Num. $= 0$

$x =$ _____

Multiplicity: ____

EVEN ODD

[TOUCH | CROSS]

4) y-int → Compute $f(0)$: _____

5) Vertical Asymptote(s) → Set Denom. = 0

$x =$ _____

6) Horizontal/Slant Asymptote(s)

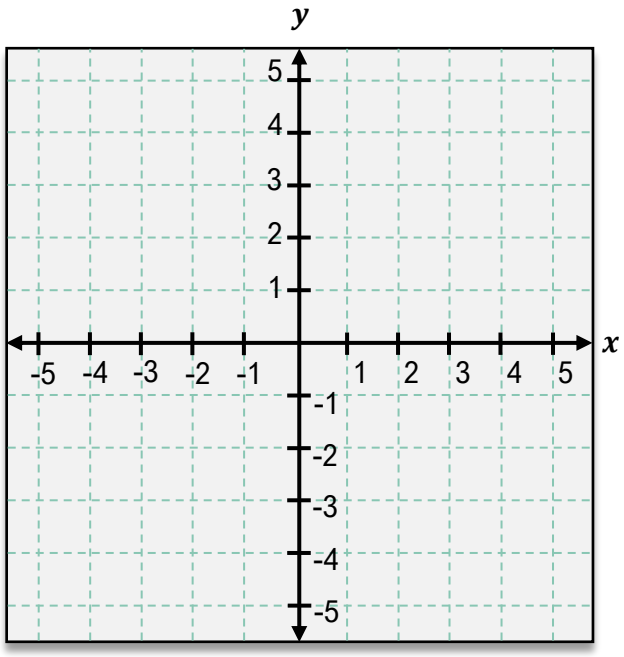
Deg. Num. [$<$ | $=$] Deg. Denom

$y =$ [0 | divide lead coeff] → _____

7) Determine intervals & plot a point in each

x				
$f(x)$				

8) Connect & draw approaching asymptotes



TOPIC: GRAPHING RATIONAL FUNCTIONS

PRACTICE: Graph the rational function.

$$f(x) = \frac{x+3}{x^2+5x+6}$$

TO GRAPH

1) Factor & find Domain → Set Denom. = 0

$\{x|x \neq$ _____$\}$

2) Holes → a. Set Common Factor(s) = 0:____

b. Put in Lowest Terms

3) x-int(s) & behavior→ Set Num. $= 0$

$x =$ _____

Multiplicity: ____

EVEN ODD

[TOUCH | CROSS]

4) y-int → Compute $f(0)$: _____

5) Vertical Asymptote(s) → Set Denom. = 0

$x =$ _____

6) Horizontal/Slant Asymptote(s)

Deg. Num. [$<$ | $=$] Deg. Denom

$y =$ [0 | divide lead coeff] → _____

7) Determine intervals & plot a point in each

x			
$f(x)$			

8) Connect & draw approaching asymptotes