TOPIC: COMBINATORICS

Fundamental Counting Principle

- You'll be asked to find the number of total possible outcomes when faced with multiple options for multiple things.

3 shirts ___ 4 pants = ____ outfits

Fundamental Counting Principle

If there are: m possible choices for one thing / outcomes of one event,

… and n possible choices for another thing / outcomes of another event,

there are _________ TOTAL possible choices for BOTH things / outcomes of BOTH events.

EXAMPLE

(A) A menu lists 4 appetizers & 6 entrees. How many different meals with both an appetizer & an entree do you have to choose from?

_______ × _______

# of choices for 1st thing — # of choices for 2nd thing

(B) How many possible outcomes are there if you flip a coin & roll a six-sided die?

_______ × _______

(C) How many different outfits can be made from 4 shirts, 5 pairs of pants, & 3 pairs of shoes?

_______ × _______ × _______

- When faced with MORE than two things, just continue to ___________ by the number of options of each thing.

TOPIC: COMBINATORICS

PRACTICE How many possible outcomes are there if you roll 5 dice?

PRACTICE How many options are there for license plates with any 3 letters (A-Z) followed by any 3 numbers (0-9)?

PRACTICE Phone numbers are 10 digits long. How many possible phone numbers are there if the 1st & 4th numbers can't be 0?

____ ____ ____ - ____ ____ ____ - ____ ____ ____ ____

TOPIC: COMBINATORICS

Introduction to Permutations

- **Permutations** are a way to arrange a number of things in a specific _________ where each thing occurs only once.
  - For example, the different ways to wear 5 shirts over 5 days are permutations, because they are in ***order***.

Monday Tuesday Wednesday Thursday Friday

_____ × _____ × _____ × _____ × _____

# of options for 1st day **# of options for 2nd day** . . . 3rd . . . 4th . . . 5th

EXAMPLE How many different ways could you choose to get dressed over 5 days if you instead had 8 shirts?

_____ × _____ × _____ × _____ × _____

- The formula for the number of permutations of r objects out of n total options is:
  - ${}_nP_r$ or $P(n, r)$ can be read as: "*number of permutations of* n *things taken* r *at a time*"

New

$${}_nP_r = \frac{n!}{(n-r)!}$$

(Permutations)

EXAMPLE

(A) A teacher is choosing a line leader & a door holder from her class of 25 students. How many ways are there for the teacher to choose these positions?

(B) On your homework, there are 10 fill-in-the-blank questions with a word bank of 14 words. If you can only use a word once, how many possible ways could you answer these 10 questions?

- Solve permutation problems easily by rewriting the numerator to ___________ the denominator.

TOPIC: COMBINATORICS

PRACTICE A student formed a club at their school. They have 13 members, and need to elect a president, vice president, and treasurer. How many ways are there to fill these officer positions?

PRACTICE Emily is organizing her closet. She has 15 shirts left to hang but has space in one section for 6 shirts. How many ways could she hang shirts in that section?

PRACTICE Evaluate the given expression. ${}_9P_4$

TOPIC: COMBINATORICS

Permutations of Non-Distinct Objects

- When finding permutations, you may be faced with multiple objects that are the _______ or ***non-distinct.***

Recall — Permutations of *Distinct* Objects

How many outcomes are there if you draw all marbles out of a bag of 6 different colored marbles?

n = total # of objects
r = # of selected objects

$${}_nP_r = \frac{n!}{(n-r)!}$$

n = total # of objects
r = # of selected objects

New — Permutations of *Non-Distinct* Objects

How many outcomes are there if you draw all marbles out of a bag of 4 blue & 2 red marbles?

$$\frac{n!}{r_1!\,r_2!\,r_3!\,\ldots}$$

n = total # of objects
r_1 = # of same objects of type 1
r_2 = # of same objects of type 2
r_3 = # of same objects of type 3

- For permutations with non-distinct objects, rewrite numerator to cancel ___________ factorial in the denominator.

EXAMPLE

(A) How many different 8-digit codes can be made from 5 zeros & 3 ones?

(B) How many ways are there to arrange the letters of BANANA?

TOPIC: COMBINATORICS

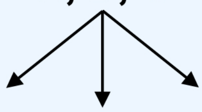
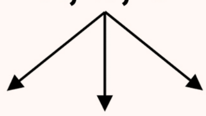
PRACTICE You want to arrange the books on your bookshelf by color. How many different ways could you arrange 12 books if 4 of them have a blue cover, 3 are yellow, and 5 are white?

PRACTICE How many ways are there to arrange the letters in the word CALCULUS?

TOPIC: COMBINATORICS

Permutations vs. Combinations

- In math, permutations & combinations are different ways by which we organize objects.
  - Distinguish between the two by considering whether or not the __________ of objects matters.

Recall **Permutations**	New **Combinations**
Permutations of 2 letters from **A, B, C** 	Combinations of 2 letters from **A, B, C** 
• A way to **[GROUP | ARRANGE]** objects • Order **[DOES | DOES NOT]** matter	• A way to **[GROUP | ARRANGE]** objects • Order **[DOES | DOES NOT]** matter

EXAMPLE Determine if the problem involves a **permutation** or **combination**. Do not solve.

(A) An ice cream shop has 32 flavors. You need to pick 2 flavors to blend into a milkshake. How many possible ways can you select your flavors?

[PERMUTATION | COMBINATION]

(B) How many ways could a photographer line up the members of a family of 5?

[PERMUTATION | COMBINATION]

(C) How many different teams of 4 people can be formed from a group of 9 people?

[PERMUTATION | COMBINATION]

TOPIC: COMBINATORICS

Combinations

- To calculate the number of **combinations** of $\boldsymbol{r}$ objects out of a group of $\boldsymbol{n}$, divide the **permutations** by ___
  - $_nC_r$ or $C(n,r)$ can be read as: "*number of combinations of* $\boldsymbol{n}$ *things taken* $\boldsymbol{r}$ *at a time*"

Recall: Permutations	New: Combinations
Permutations of 2 letters from **A, B, C** AB AC BC BA CA CB	Combinations of 2 letters from **A, B, C** AB AC BC
$_nP_r = \frac{n!}{(n-r)!}$ → $_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!}$	$_nC_r = \frac{n!}{(n-r)! \cdot \_\_\_}$ → $_3C_2 = \frac{3!}{(3-2)! \cdot 2!} = \frac{3!}{\_\_\_}$

- Recall: When there are multiple factorials in denominator, rewrite numerator to cancel *highest* factorial in denominator.

EXAMPLE

$(\boldsymbol{A})$ An ice cream shop has 32 flavors. You need to pick 2 flavors to blend into a milkshake. How many possible ways can you select your flavors?

$(\boldsymbol{B})$ How many different teams of 4 people can be formed from a group of 9 people?

TOPIC: COMBINATORICS

PRACTICE From a class of 28 students, in how many ways could a teacher select 4 students to lead the class discussion?

PRACTICE Evaluate the given expression. ${}_{11}C_7$