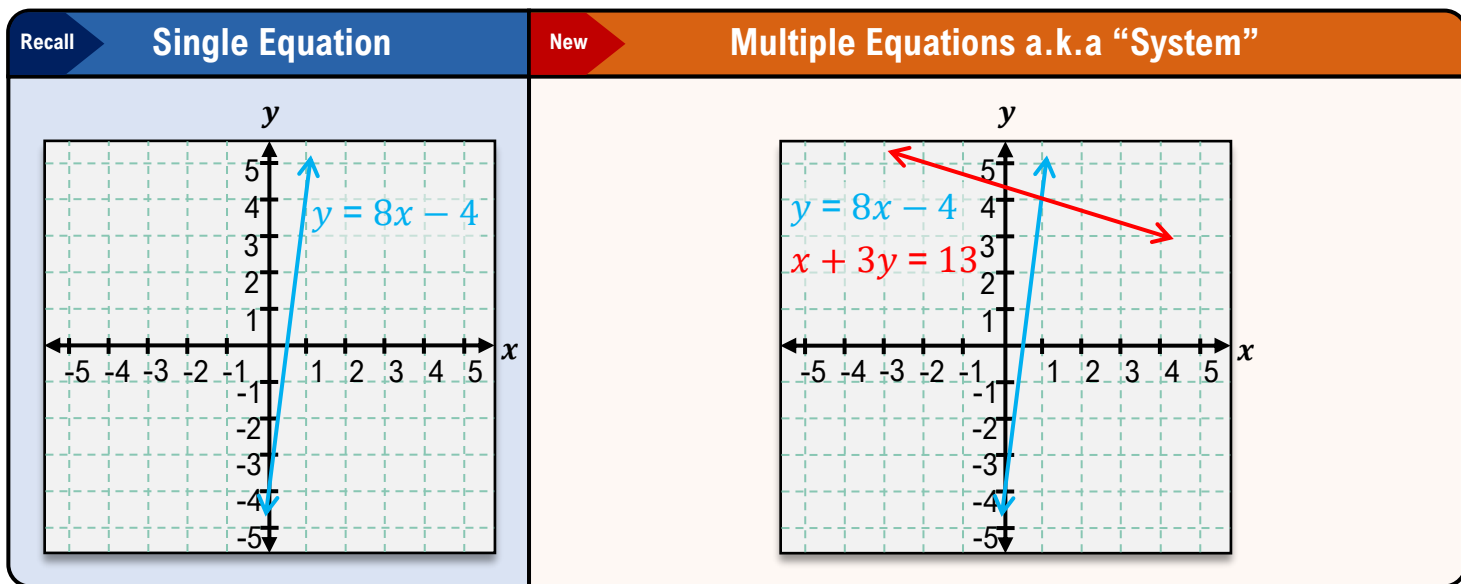


# **TOPIC: SYSTEMS OF LINEAR EQUATIONS** **Introduction to Systems of Linear Equations**

- ◆ Previously, we solved & plotted *single* equations on graphs. Now we'll look at **Systems of ( ) Equations**.
  - To solve, find  $(x, y)$  which satisfy \_\_\_\_\_ equations. Graphically, this is where the lines \_\_\_\_\_.



## **EXAMPLE**

Determine if each point is a solution to the equation.

- |           |           |          |
|-----------|-----------|----------|
| (A)       | (B)       | (C)      |
| $(-2, 0)$ | $(0, -4)$ | $(1, 4)$ |

Solution: [ 1 | MANY ] point(s)  
 satisfying [ 1 | ALL ] line(s)

Determine if each point is a solution to the system of equations.

- |           |           |          |
|-----------|-----------|----------|
| (A)       | (B)       | (C)      |
| $(0, -4)$ | $(-2, 5)$ | $(1, 4)$ |

Solution: [ 1\* | MANY ] point(s)  
 satisfying [ 1 | ALL ] line(s)

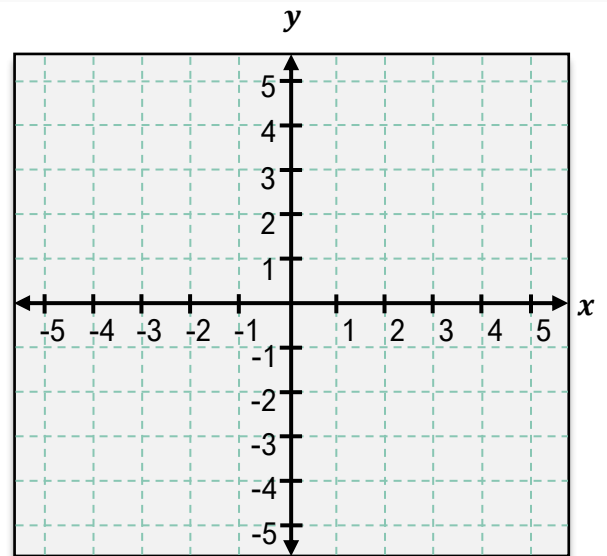
\* True for *most* problems, but there are other types of solutions

## TOPIC: SYSTEMS OF LINEAR EQUATIONS

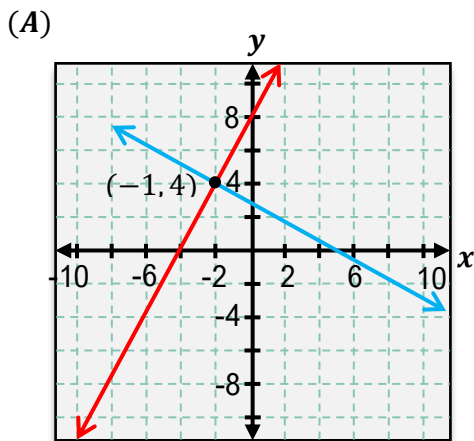
**EXAMPLE** Graph the system of equations. Identify the intersection point and verify it is a solution to both equations.

$$y = 2x + 3$$

$$y = x + 4$$

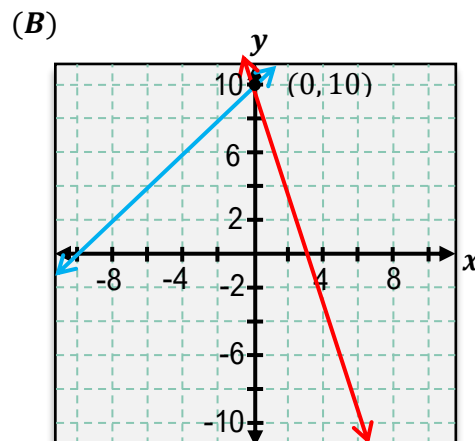


**EXAMPLE** Match each system of equations to its graph and solution.



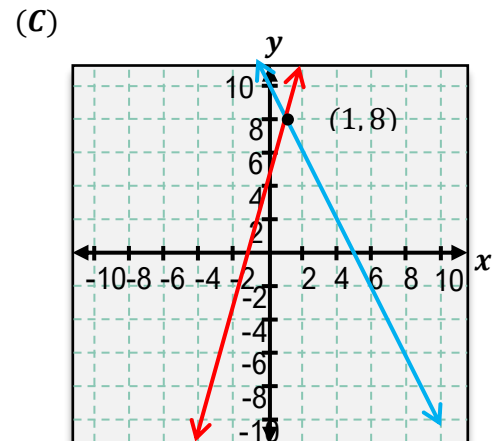
(1)

$$y = 3x + 5$$
$$y = -2x + 10$$



(2)

$$y = 4x + 8$$
$$2x + 3y = 10$$



(3)

$$3x + y = 10$$
$$-x + y = 10$$

## TOPIC: SYSTEMS OF LINEAR EQUATIONS

### Solving Systems of Linear Equations – Substitution

◆ One way to solve systems of equations *without* graphing is by \_\_\_\_\_ one equation into another.

#### EXAMPLE

Solve the system of equations using substitution.

$$y = 7x - 14$$

$$2x - y = 4$$

#### HOW TO: Solve Systems of Equations – Substitution

- 1) Choose easiest equation to isolate  $x$  or  $y$  as **(A)**
- 2) Solve **(A)** for  $x$  OR  $y$
- 3) Substitute **(A)** into **(B)**, then solve **(B)**
- 4) Plug value from **3)** back into *either* eq'n, then solve
- 5) Check answer by plugging values into both eqn's

#### PRACTICE

Use substitution to solve the following system of linear equations.

$$4x + y = 1$$

$$x - y = 4$$

## TOPIC: SYSTEMS OF LINEAR EQUATIONS

**PRACTICE** Use substitution to solve the following system of linear equations.

$$\begin{aligned}4x + 2y &= 7 \\ x + 5y &= 4\end{aligned}$$

### HOW TO: Solve Systems of Equations – Substitution

- 1) Choose easiest equation to isolate  $x$  or  $y$  as (A)
- 2) Solve (A) for  $x$  OR  $y$
- 3) Substitute (A) into (B), then solve (B)
- 4) Plug value from 3) back into *either* eq'n, then solve
- 5) Check answer by plugging values into both eqn's

## TOPIC: SYSTEMS OF LINEAR EQUATIONS

### Solving Systems of Linear Equations – Elimination

- ◆ Another way to solve systems of equations is by \_\_\_\_\_ the equations & \_\_\_\_\_ a variable.
- If not asked to use a specific method, use this when equations are in standard form or have large coefficients.  
( $Ax + By = C$ )

Recall	Substitution	New	Elimination
	$y = 5x - 3$ $x = 2$		$x + y = 1$ $-x + y = 5$

#### EXAMPLE

Solve the system of equations using elimination.

$$3x + 2y = 1$$

$$-x + y = 3$$

#### HOW TO: Solve Systems of Equations - Elimination

- 1) Write *BOTH* equations in the form  $Ax + By = C$ , aligning coeff's vertically on top of each other
- 2) Multiply eq'n(s) by # (+ or -) so  $x$  or  $y$  coeff's are \_\_\_\_\_ with \_\_\_\_\_ signs
- 3) \_\_\_\_\_ equations vertically to eliminate one variable, then solve for other
- 4) Plug value from **3)** back into *either* eq'n, then solve
- 5) Check answer by plugging values into both eqn's

## TOPIC: SYSTEMS OF LINEAR EQUATIONS

### How to Multiply Equations in Elimination Method

◆ To determine what # to multiply by in Step 2), look at the coefficients of each equation.

Elimination Method – What to Multiply Equation(s) by to Eliminate Variable				
If coefficients of $x$ or $y$ are...	Equal with <i>OPPOSITE</i> sign	Equal with <i>SAME</i> sign	Factors of each other (Evenly divisible)	Anything Else
Multiply...	Nothing! Just add	Either eq'n by $-1$	Eq'n with smaller coeff's by quotient	Each eq'n by <i>other</i> coeff (+ or -)
EXAMPLE	$7x + 13y = 12$ $-7x + 2y = 18$	$5x + 7y = 17$ $6x + 7y = 12$	$12x - 5y = 24$ $3x - 2y = 6$	$6x + 2y = -10$ $-4x - 3y = 15$

### HOW TO: Solve Systems of Equations - Elimination

- 1) Write *BOTH* equations in the form  $Ax + By = C$ , aligning coeff's vertically on top of each other
- 2) Multiply eq'n(s) by # (+ or -) so  $x$  or  $y$  coeff's are **EQUAL** with **OPPOSITE** signs
- 3) **ADD** equations vertically to eliminate one variable, then solve for other
- 4) Plug value from 3) back into *either* eq'n, then solve
- 5) Check answer by plugging values into both eq'ns

## TOPIC: SYSTEMS OF LINEAR EQUATIONS

**EXAMPLE** Without *fully* solving, multiply one or both equation(s) by an appropriate factor to cancel out a variable.

Elimination Method – What to Multiply Equation(s) by to Eliminate Variable				
If coefficients of $x$ or $y$ are...	Equal with <i>OPPOSITE</i> signs	Equal with <i>SAME</i> signs	Factors of each other (Evenly divisible)	Anything Else
Multiply...	Nothing! Just add	Either eq'n by $-1$	Eq'n with smaller coeff's by quotient	Each eq'n by <i>other</i> coeff (+ or $-$ )

$$\begin{aligned}2x + 3y &= 1 \\ x - y &= 3\end{aligned}$$

**EXAMPLE** Without *fully* solving, multiply one or both equation(s) by an appropriate factor to cancel out a variable.

$$\begin{aligned}5x + 3y &= 10 \\ -7x + 5y &= 15\end{aligned}$$

## TOPIC: SYSTEMS OF LINEAR EQUATIONS

**PRACTICE** Use the elimination method to solve the following system of linear equations.

$$\begin{aligned}2x + y &= 1 \\ 3x - y &= 4\end{aligned}$$

### HOW TO: Solve Systems of Equations – Elimination

- 1) Write *BOTH* equations in the form  $Ax + By = C$ , aligning coeff's vertically on top of each other
- 2) Multiply eq'n(s) by # (+ or -) so  $x$  or  $y$  coeff's are **EQUAL** with **OPPOSITE** signs
- 3) **ADD** equations vertically to eliminate one variable, then solve for other
- 4) Plug value from 3) back into *either* eq'n, then solve
- 5) Check answer by plugging values into both eqn's

**PRACTICE** Use the elimination method to solve the following system of linear equations.

$$\begin{aligned}10x - 4y &= 5 \\ 5x - 4y &= 1\end{aligned}$$



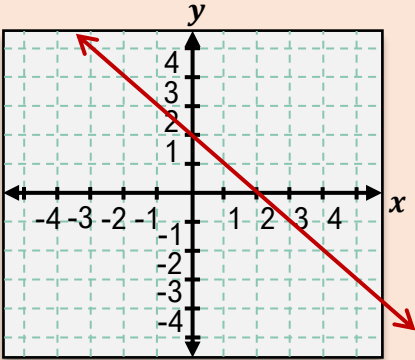
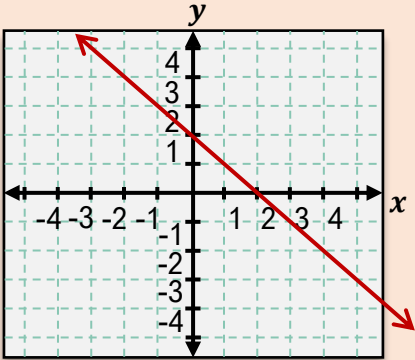
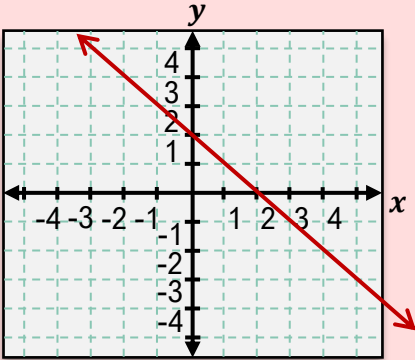
## TOPIC: SYSTEMS OF LINEAR EQUATIONS

### Classifying Systems of Linear Equations

Like linear equations, systems can be put in 3 **categories** based on the \_\_\_\_\_ of solutions they have.

#### EXAMPLE

Solve each system of equations. Then graph and categorize the system.

Consistent		Inconsistent
Independent	Dependent	
$y = 3$ $x + y = 2$	$-x - y = -2$ $x + y = 2$	$y = -x + 3$ $x + y = 2$
		
Lines are _____ $[1 \mid \infty \mid 0]$ Solutions	Lines are _____ $[1 \mid \infty \mid 0]$ Solutions	Lines are _____ $[1 \mid \infty \mid 0]$ Solutions

**TOPIC: SYSTEMS OF LINEAR EQUATIONS**

**PRACTICE**

Solve the following system of equations. Classify it as CONSISTENT (INDEPENDENT or DEPENDENT) or INCONSISTENT.

$$\begin{aligned}y &= 5x - 17 \\ 15x - 3y &= 51\end{aligned}$$

**PRACTICE**

Solve the following system of equations. Classify it as CONSISTENT (INDEPENDENT or DEPENDENT) or INCONSISTENT.

$$\begin{aligned}2x + 8y &= 7 \\ x + 4y &= 19\end{aligned}$$