Introduction to Probability

- lacktriangle How likely an event is to happen is called the **probability** of the event, written as P(event)
 - ▶ In general, $P(event) = \frac{\# of times event occurs}{TOTAL}$

Theoretical Probability

Empirical (Experimental) Probability

Toss # 1 2 3
Result T H H

P(heads) = ----

- P(heads) = ----
- ▶ Based on what [COULD | DID] happen
- ► Calculated [BEFORE | AFTER] events occur

 $P(event) = \frac{\text{# of outcomes with event}}{\text{# of TOTAL possible outcomes}}$

▶ Based on what [COULD | DID] happen

▶ Calculated [BEFORE | AFTER] events occur

$$P(event) = \frac{\# \ of \ times \ event \ occured}{TOTAL \ \# \ of \ trials}$$

EXAMPLE

When rolling a six-sided die, what is the probability of rolling a number greater than 3?

EXAMPLE

The table below shows the results of rolling a six-sided die 10 times. Given that data, what is the probability of rolling a number greater than 3?

Roll	1	2	3	4	5	6	7	8	9	10
Outcome	6	4	2	5	5	5	6	1	4	5

- ◆ You may see the possible outcomes of an event expressed as a set, referred to as a sample space.
 - ► For example, the **sample space** of flipping a coin is *S* = {______}

PRACTICE

Given the data below, determine the probability that a person randomly selected from Group 1 will be wearing jeans.

	Group 1	Group 2	Group 3
Wearing Jeans	68	27	17
Not wearing jeans	63	36	89

PRACTICE

In your coin purse, you have 3 quarters, 4 nickels, & 2 dimes. If you pick a coin at random, what is the probability that it will be a quarter?

EXAMPLE

When playing a particular lottery game, you must choose 5 numbers between 1 and 40. You win if the 5 numbers you choose match those drawn in the lottery. What is the probability that you will win if you purchase...

(**A**) 1 lottery ticket?

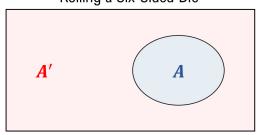
Recall $P(event) = \frac{\# \ of \ outcomes \ with \ event}{\# \ of \ TOTAL \ outcomes}$ (Probability) $nC_r = \frac{n!}{(n-r)! \cdot r!}$ (Combinations)

(**B**) 50 different lottery tickets?

Complementary Events

- ◆ All outcomes where an event A does **NOT** occur is the **complement** of A (written as _____, ____ or ____).
 - ▶ The total probability of ALL possible events is ALWAYS _____.

Rolling a Six-Sided Die



EXAMPLE

(A) When rolling a six-sided die, probability that you will roll a 4?

What is the probability that you will NOT roll a 4?

Recall
$$P(event) = \frac{\# of \ outcomes \ with \ event}{\# of \ TOTAL \ outcomes}$$

(B) When drawing a single card from a standard deck of 52, what it the probability that you will NOT draw a queen?

PRACTICE

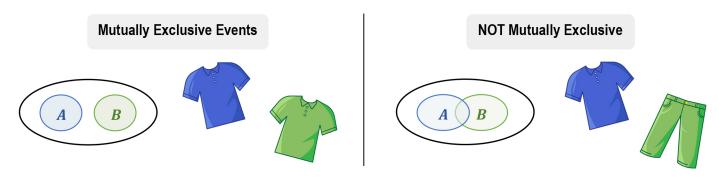
When drawing a marble out of a bag of red, green, and yellow marbles 8 times, a red or yellow marble is drawn 6 times. What is the probability of drawing a green marble?

PRACTICE

A weatherman states that the probability that it will rain tomorrow is 10%, or 0.1, & the probability that it will snow is 25%, or 0.25. What is the probability that it will not rain or snow?

Probability of Mutually Exclusive Events

◆ Events which **CANNOT** happen at the same time are **mutually exclusive**.



EXAMPLE

Identify whether each set of events is mutually exclusive or not.

(A) Getting heads when flipping a coin vs getting tails

(B) Getting a 6 when rolling a die vs getting a number higher than 3

Events [ARE | ARE NOT] mutually exclusive

Events [ARE | ARE NOT] mutually exclusive

◆ To find the probability of any one of multiple mutually exclusive events occurring, _____ the probability of each.

▶ A ∪ B means any event in A OR B

 $P(A \cup B) = \underline{\hspace{1cm}}$

EXAMPLE

You roll a six-sided die. What is the probability of getting a 3 OR a 5?

Recall $P(event) = \frac{\# of \ outcomes \ with \ event}{\# of \ TOTAL \ outcomes}$

PRACTICE

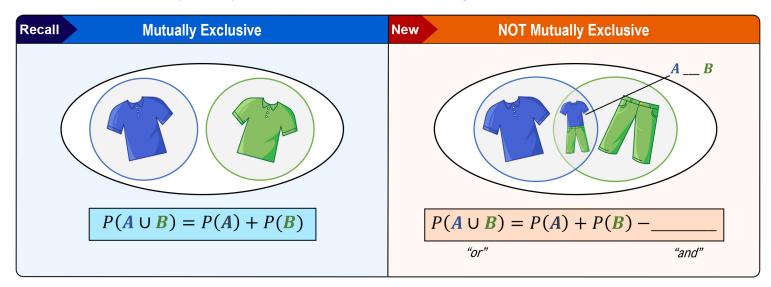
If a single card is randomly selected from a deck of cards, what is the probability of selecting an ace or a king?

PRACTICE

For two mutually exclusive events A and B, compute $P(A \cup B)$ if P(A) = 0.15 and P(B) = 0.32

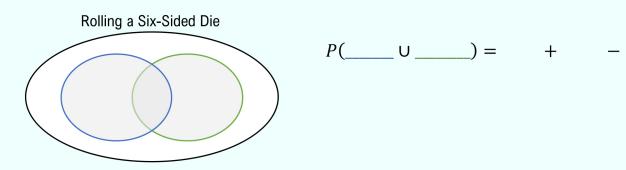
Probability of Non-Mutually Exclusive Events

- ◆ For events which are **NOT** mutually exclusive, there is overlap in which both events can occur at the same time.
 - We must subtract the probability of the ______, so it doesn't get counted twice.



EXAMPLE

When rolling a six-sided die, what is the probability of rolling a number greater than 3 OR an even number?



♦ The equation for $P(A \cup B)$ is the same for all events, but for mutually exclusive events, $P(A \cap B)$ is always ____.

EXAMPLE

The table below shows the outfits of 300 observed people on a given day. Of one person randomly selected from this group, what is the probability that they will be wearing shorts or a green shirt?

	Wearing a red shirt	Wearing a blue shirt	Wearing a green shirt	Total
Wearing Pants	68	27	17	112
Wearing Shorts	63	36	89	188
Total	131	63	106	300

PRACTICE

A card is drawn from a standard deck of 52 cards. What is the probability that the card is a diamond or a king?

Probability of Multiple Independent Events

◆ Events which do **NOT** depend on each other at all are called **Independent Events**.

EXAMPLE

Identify whether each set of events is independent or dependent.

(A) Getting tails on the first toss of a coin Getting tails on the second toss of a coin



(B) Drawing and keeping a blue marble from a bag Drawing a blue marble again



[INDEPENDENT | DEPENDENT]

[INDEPENDENT | DEPENDENT]

◆ For independent events, find the probability of event *A AND* event *B* occurring by _____ their probabilities.

$$P(A \cap B) = P(A) \underline{\hspace{1cm}} P(B)$$

EXAMPLE

Find the probability of each set of events.

(A) Getting heads on two consecutive coin flips

(B) Rolling an even number on the first roll of a six-sided die and rolling a 3 on the second roll

ullet For any number of **independent events**, multiply ____ probabilities to find $P(A \cap B \cap C \cap ...)$.

PRACTICE

The spinner below has 6 equal regions. Find the probability of landing on yellow for the first spin and not landing on yellow on the second spin.



PRACTICE

The spinner below has 6 equal colored regions numbered 1-6. Find the probability of stopping on yellow for the first spin, stopping on an even number on the second spin, and stopping on blue or red on the third spin.

