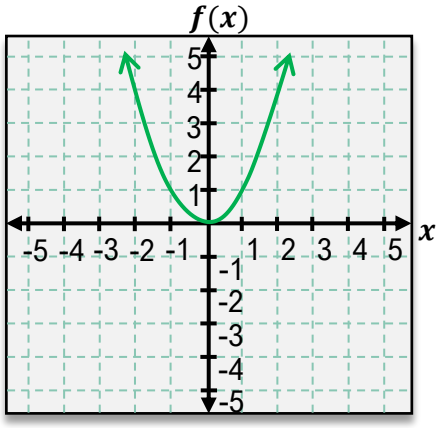
TOPIC: ASYMPTOTES

Introduction to Asymptotes

- To graph a rational function, you need to identify **asymptotes**: line that a curve approaches but does ***NOT*** _____.

Polynomial Functions

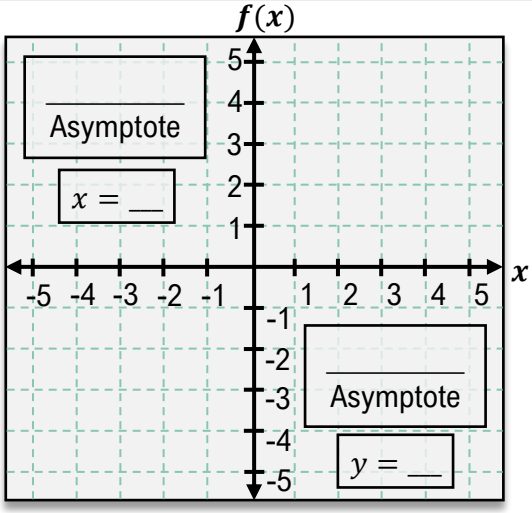


As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to \infty$

As $x \to 0$, $f(x) \to 0$

Rational Functions



As $x \to \infty$, $f(x) \to$ _____

As $x \to -\infty$, $f(x) \to$ _____

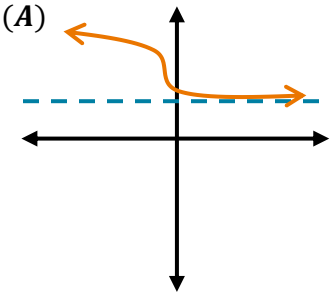
(from _____) As $x \to 0^+$, $f(x) \to$ _____

(from _____) As $x \to 0^-$, $f(x) \to$ _____

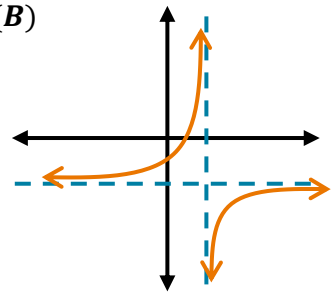
x	$f(x) = \frac{1}{x}$
-10	$-\frac{1}{10}$
…	
-1	-1
…	
0.01	
0.1	
1	
2	
3	
…	
10	
…	
100	

- A rational function may have *none, one,* or *multiple* asymptotes, represented with dashed lines.

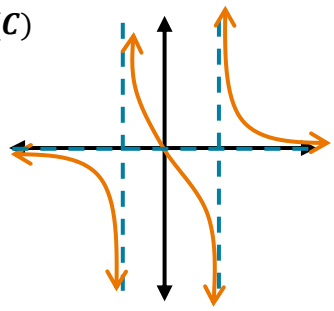
(A)



(B)

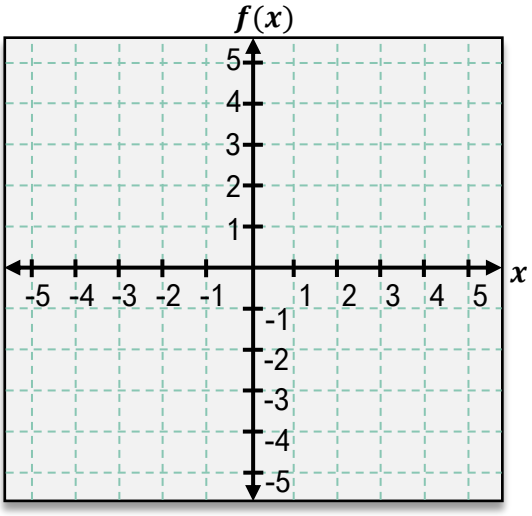


(C)



TOPIC: ASYMPTOTES

PRACTICE: Sketch the graph of the function $f(x) = \frac{1}{x^2}$. Identify the asymptotes on the graph.



Vertical Asymptote: $x =$_____

Horizontal Asymptote: $y =$_____

x	$f(x) = \frac{1}{x^2}$
-10	
...	
-1	
...	
-0.1	
...	
0.1	
1	
2	
3	
...	
10	

TOPIC: ASYMPTOTES

Determining Vertical Asymptotes

- To find vertical asymptotes, put the function in _____, then set denominator = ___ & solve for x.

$f(x) = \frac{x+2}{(x+2)(x-3)}$

Set Denominator = 0 & solve for x

Domain: $\{x | x \neq -2, x \neq 3\}$

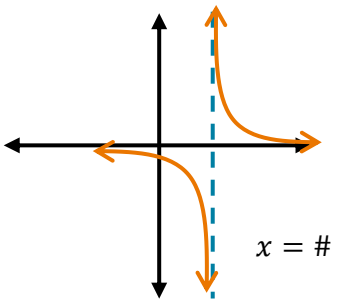
$f(x) = \frac{x+2}{(x+2)(x-3)}$

Lowest Terms

$f(x) =$ _____

Set Denominator = 0 & solve for x

Vertical Asymptote: $x =$ ____



- Recall: Put function in lowest terms by factoring and cancelling common factors.

EXAMPLE: Find the vertical asymptote(s) of each function.

(A) $f(x) = \frac{2}{2x+6}$

VA(s): $x =$ _____

(B) $f(x) = \frac{1}{x^2-9}$

VA(s): $x =$ _____

$x =$ _____

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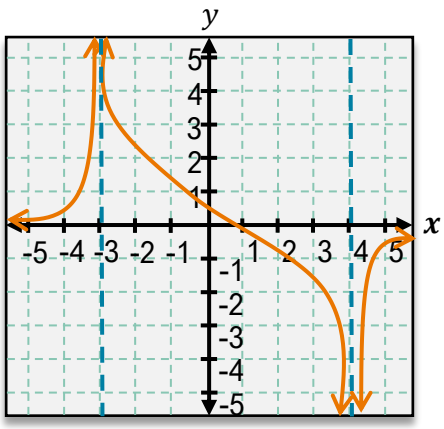
PRACTICE: Based only on the vertical asymptotes, which of the following graphs could be the graph of the given function?

$f(x) = \frac{x^2-4x}{x^2-x-12}$

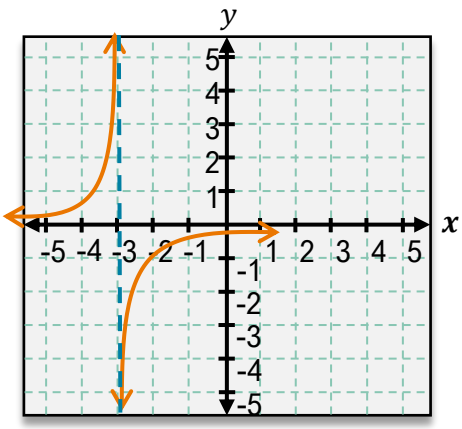
VA(s): x = _____

x = _____

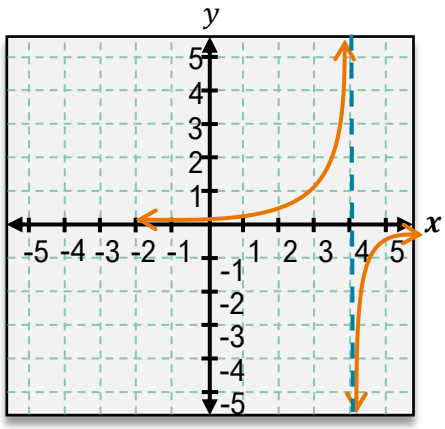
$(\boldsymbol{A})$



$(\boldsymbol{B})$



$(\boldsymbol{C})$

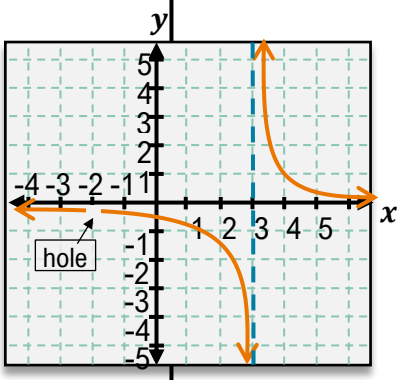


TOPIC: ASYMPTOTES

Determining Removable Discontinuities (Holes)

- To find **holes** in the graph of a rational function, factor, then set _____ $= 0$ & solve for x.
  - Holes (removable discontinuities) are represented on graphs as open circles.

Vertical Asymptotes	Holes
$f(x) = \frac{x+2}{(x+2)(x-3)}$	$f(x) = \frac{x+2}{(x+2)(x-3)}$
Cancel common Factor *(lowest terms)*	Set common factor = 0
$f(x) = \frac{1}{x-3} = 0$	_____ = __
Set Denom. = 0 & solve for x	Solve for x
Vertical Asymptote: $x = 3$	Hole: x = ____



EXAMPLE: Find any holes in the graph of the given function.

(A) $f(x) = \frac{x+3}{x^2+4x+3}$

Hole: x = _____

(B) $f(x) = \frac{x^2-1}{x-1}$

Hole: x = _____

PRACTICE: Find all vertical asymptotes and holes of each function.

(A) $f(x) = \frac{-5x}{(2x-3)^2}$

Hole(s): x = _____ VA(s): x = _____

x = _____ x = _____

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PRACTICE**:** Find all holes and vertical asymptotes of each function.

($\boldsymbol{B}$)

$$f(x) = \frac{x^2-2x}{2x^3-x^2-6x}$$

Hole(s): x = _____ VA(s): x = _____

x = _____ x = _____

($\boldsymbol{C}$)

$$f(x) = \frac{x^2+10x+25}{2x^2+8x-10}$$

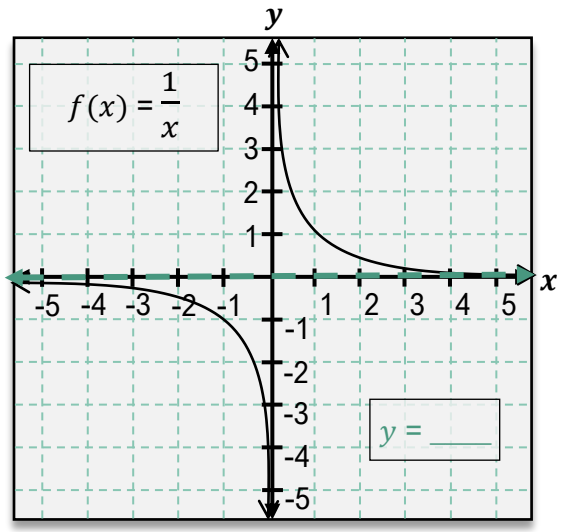
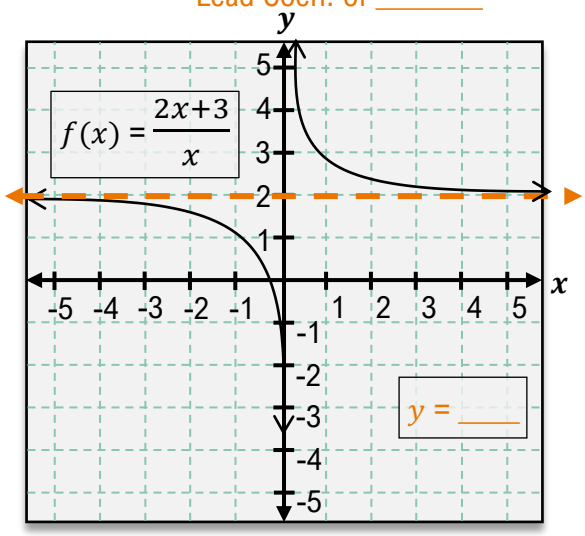
Hole(s): x = _____ VA(s): x = _____

x = _____ x = _____

TOPIC: ASYMPTOTES

Horizontal Asymptotes

- Vertical Asymptotes affect the domain of a rational function, but **Horizontal Asymptotes** affect the _____.
  - The horizontal asymptote depends on the degrees of the _____ & _____.

If Degree Num. < Degree Denom.	Degree Num. = Degree Denom.
$y =$ ____	$y =$ Lead Coeff. of _____ / Lead Coeff. of _____
	

EXAMPLE: Identify the horizontal asymptote of each function.

(A) $f(x) = \frac{4x^2}{-x^3-5x+9}$

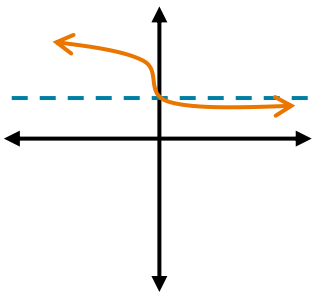
Deg. Num. [< | =] Deg. Denom.

HA: $y =$_____

(B) $f(x) = \frac{2x^2}{3x^2+x-1}$

Deg. Num. [< | =] Deg. Denom.

HA: $y =$_____



- The graph of a rational function *may* intersect a horizontal asymptote.

TOPIC: ASYMPTOTES

PRACTICE: Find the horizontal asymptote of each function.

$(\boldsymbol{A})$ $f(x) = \frac{-5x}{(2x+3)^2}$

Deg. Num. [$<$ | $=$] Deg. Denom.

HA: y = _____

$(\boldsymbol{B})$ $f(x) = \frac{8x^2+1}{2x^2-x-6}$

Deg. Num. [$<$ | $=$] Deg. Denom.

HA: y = _____

$(\boldsymbol{C})$ $f(x) = \frac{x^2+4x}{2x^3+8x^2}$

Deg. Num. [$<$ | $=$] Deg. Denom.

HA: y = _____