TOPIC: SEQUENCES

Introduction to Sequences

◆ A **Sequence** is a LIST of numbers in a specific ____________.

$$\{2,4,6,8,\_\_\_\}$$

- The _______________ in a sequence are called **Terms** (a.k.a. "elements" or "members").
- Sequences can be ***finite*** (________ after a certain number) or ***infinite*** (go on __________).

EXAMPLE Find the 5th term in each sequence & identify if the sequence is *finite* or *infinite*.

(A) $\{3,6,9,12,\_\_\_,18\}$

[FINITE | INFINITE]

(B) $\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{}{},\ldots\}$

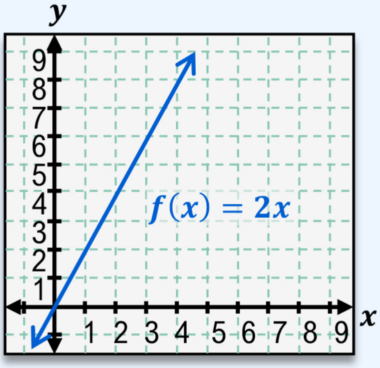
[FINITE | INFINITE]

◆ Sequences are like *functions*; they can follow _______ or equations.

- Inputs are called **Indexes**, represented by ____, which *ALWAYS* starts at 1 and increases by 1
- Outputs are called **Terms**, represented by ____.

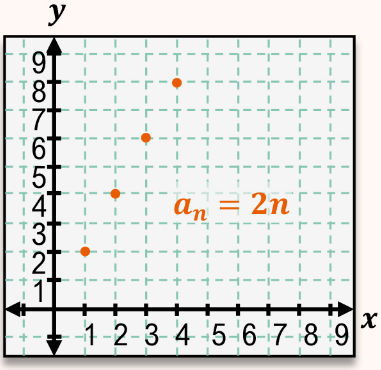
Recall — **Functions**

Inputs	x	-1	2	2.5	$\sqrt{7}$	π
Outputs	$f(x)=2x$	-2	4	5	5.29	6.28



New — **Sequences**

Indexes	n	1	2	3	4	5
Terms	$a_n=2n$					



EXAMPLE Find the first 3 terms in each sequence.

(A) $a_n = n^2$

$a_1 = \_\_\_$, $a_2 = \_\_\_$, $a_3 = \_\_\_$

(B) $a_n = \frac{1}{n+3}$

$a_1 = \_\_\_$, $a_2 = \_\_\_$, $a_3 = \_\_\_$

(C) $a_n = (-1)^n$

$a_1 = \_\_\_$, $a_2 = \_\_\_$, $a_3 = \_\_\_$

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PRACTICE The first 4 terms of a sequence are $\{\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, \ldots\}$. Continuing this pattern, find the 7th term.

PRACTICE Determine the first 3 terms of the sequence given by the general formula.

$$a_n = \frac{1}{n! + 1}$$

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Writing a General Formula

- The **General** ("explicit") **Formula** of a sequence is an equation for a_n ("general term") containing n. ($n = 1, 2, 3, \dots$)
  - To determine the general formula, find the _____________ between the numbers.

Common Patterns in General Formulas of Sequences

If sequences...	Increase by 1 or 2 or 3...	Alternate signs	Contain fractions	Increase exponentially
Formula contains...*	n or $2n$ or $3n\dots$	$(-,+,-,\dots) \rightarrow (-1)^n$ $(+,-,+,\dots) \rightarrow (-1)^{n+1}$	Fractions (top & bottom may be different)	$(\#)^n$
EXAMPLE	$\{5, 6, 7, 8, 9\}$	$\{-5, 5, -5, 5, -5\}$	$\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots\right\}$	$\{2, 4, 8, 16, 32\}$

*Note: You will often have to adjust your formula by $+, -, \times, \div$ constants to get the desired sequence.

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EXAMPLE

Given the first 4 terms of a sequence shown below, write the general formula for the n^{th} term and use it to calculate the 15th term.

$$\{\frac{1}{1\cdot 2},\frac{1}{2\cdot 3},\frac{1}{3\cdot 4},\frac{1}{4\cdot 5},\ldots\}$$

EXAMPLE

Given the first 4 terms of a sequence shown below, write the general formula for the n^{th} term and use it to calculate the 18th term.

$$\{-2,4,-6,8,-10,\ldots\}$$

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Recursive Formula

◆ Like general formulas, recursive formulas tell us how to find the n^{th} term in a sequence.

- *However*, **Recursive Formulas** show how to find $\boldsymbol{a_n}$ based on the ____________ term ($\boldsymbol{a_{n-1}}$) instead of $\boldsymbol{n}$.

Recall — General Formula

Indexes	n	1	2	3	4	5
Terms	$a_n = 2n$	$a_1 = 2$	$a_2 = 4$	$a_3 = 6$	$a_4 = 8$	$a_5 = 10$

$$a_n = 2n$$

Need [**n** | **PREVIOUS TERM**] to calculate n^{th} term

New — Recursive Formula

Indexes	n	1	2	3	4	5
Terms	$a_n = a_{n-1} + 2$	$a_1 = 2$				

$$a_n = a_{n-1} + 2$$

Need [**n** | **PREVIOUS TERM**] to calculate n^{th} term

EXAMPLE Given the recursive formula and first term of each sequence below, find the next 3 terms.

(**A**)

$$a_n = 2a_{n-1} + 3$$

$a_1 = 1$, $a_2 =$ ___ , $a_3 =$ ___ , $a_4 =$ ___

(**B**)

$$a_n = n \cdot a_{n-1}$$

$a_1 = 1$, $a_2 =$ ___ , $a_3 =$ ___ , $a_4 =$ ___

PRACTICE Write the first 6 terms of the sequence given by the recursive formula $a_n = a_{n-2} + a_{n-1}$; $a_1 = 1; a_2 = 1$