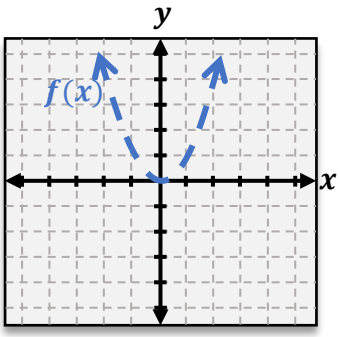
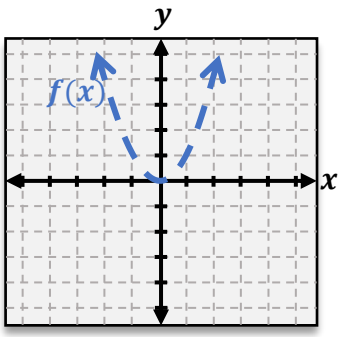
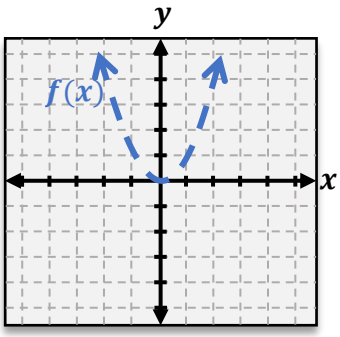
TOPIC: TRANSFORMATIONS

Intro to Transformations

- Transformations occur when a function is _____ and changes position and/or shape.

Transformations of Functions

1) Reflection (*FOLDING a function*)	2) Shift (*MOVING a function*)	3) Stretch (*SQUEEZING a function*)
		
$-f(x)$	$f(x-h)+k$	$c \cdot f(x)$

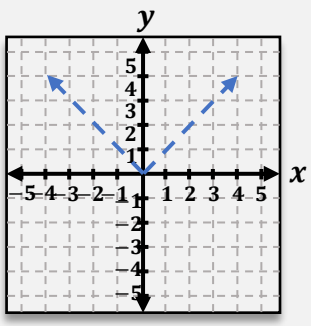
EXAMPLE: The graph of the $f(x) = |x|$ is shown below. Match the following functions $\mathbf{p(x)}, \mathbf{q(x)}$, & $\mathbf{r(x)}$ to their corresponding graphs.

Reflection	Shift	Stretch
$-f(x)$	$f(x-h)+k$	$c \cdot f(x)$

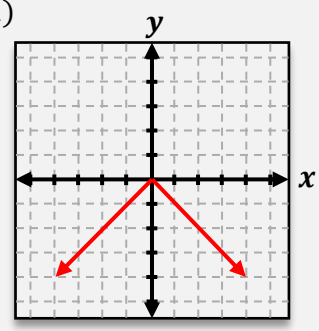
(A) $\mathbf{p(x)} = |x-3|+2$

(B) $\mathbf{q(x)} = -|x|$

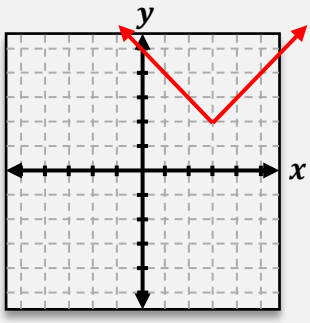
(C) $\mathbf{r(x)} = -4|x|$



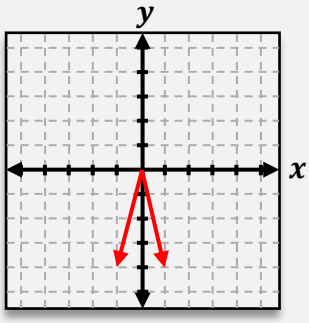
(1)



(2)



(3)

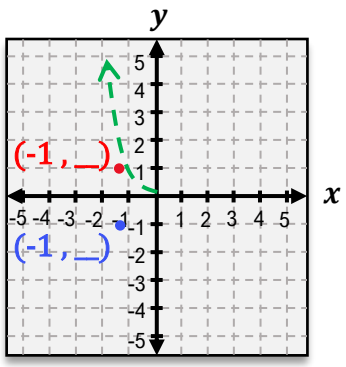
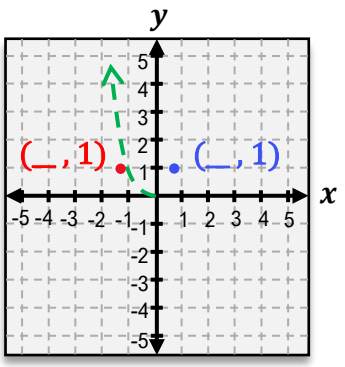


TOPIC: TRANSFORMATIONS

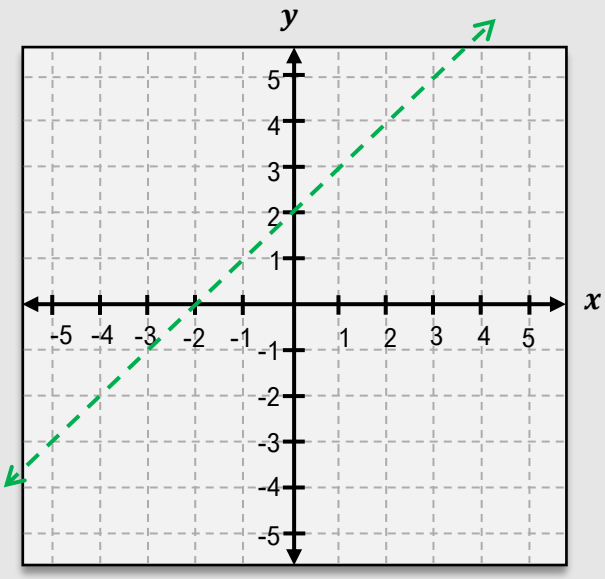
Reflections

Transformations of Functions		
Reflection	**Shift**	**Stretch**

- A reflection is a transformation where the function appears to be "_____" over the ___ -axis, or the ___ -axis.

REFLECTION OVER [X-AXIS | Y-AXIS]	REFLECTION OVER [X-AXIS | Y-AXIS]
[x's | y's] change signs	[x's | y's] change signs
	
$f(x) \rightarrow -f(x)$	$f(x) \rightarrow f(-x)$

EXAMPLE: Given $f(x) = x + 2$, let $h(x)$ be the function $f(x)$ with a ***reflection*** over the x-axis. Sketch the graph of $h(x)$, and determine which of the functions below matches the new function $h(x)$.



(A) $h(x) = x - 2$

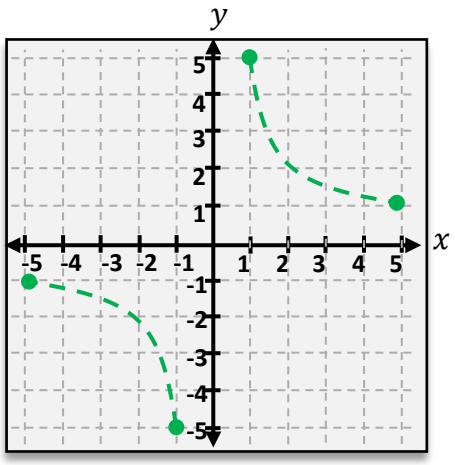
(B) $h(x) = -x + 2$

(C) $h(x) = -x - 2$

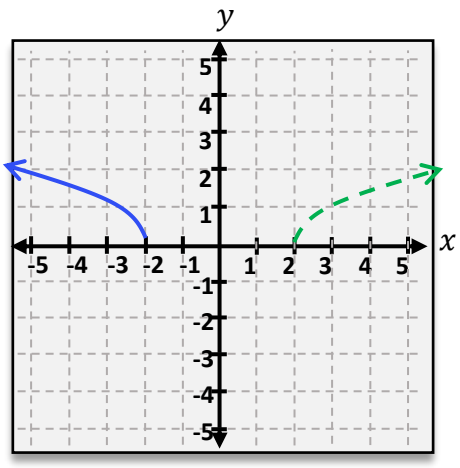
(D) $h(x) = x + 2$

TOPIC: TRANSFORMATIONS

PROBLEM: Written below is a graph of the function $f(x)$. Sketch a graph of the function $g(x)$, where $g(x)$ is the function $f(x)$ after it has been reflected over the ***x-axis***.



PRACTICE: Written below is a graph of the function $f(x) = \sqrt{x-2}$. If ***g(x)*** is a reflection of ***f(x)*** about the ***y-axis*** what is the equation for ***g(x)***?



TOPIC: TRANSFORMATIONS
Shifting A Function

Transformations of Functions		
Reflection	Shift	Stretch

- A shift occurs when a function is moved _____ and/or _____ from its original position.

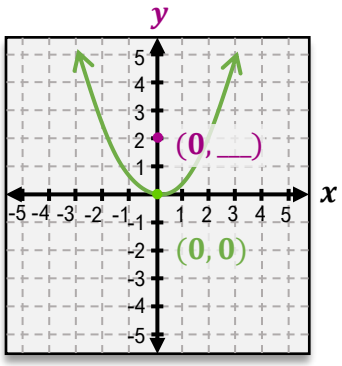
SHIFTS

General Form: $f(x) \rightarrow f(x-h)+k$

VERTICAL SHIFT ($h = 0$)

Vertical shifts change the
[x's | y's]

$f(x) \rightarrow f(x-h)+k$



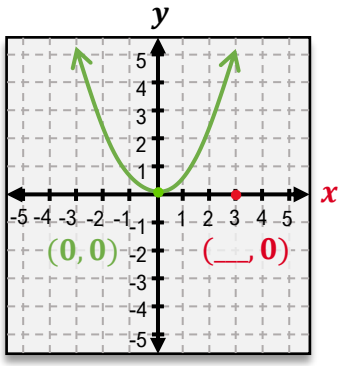
For $f(x)+k$, graph shifts **[UP | DOWN]**

For $f(x)-k$, graph shifts **[UP | DOWN]**

HORIZONTAL SHIFT ($k = 0$)

Horizontal shifts change the
[x's | y's]

$f(x) \rightarrow f(x-h)+k$



For $f(x+h)$, graph shifts **[RIGHT | LEFT]**

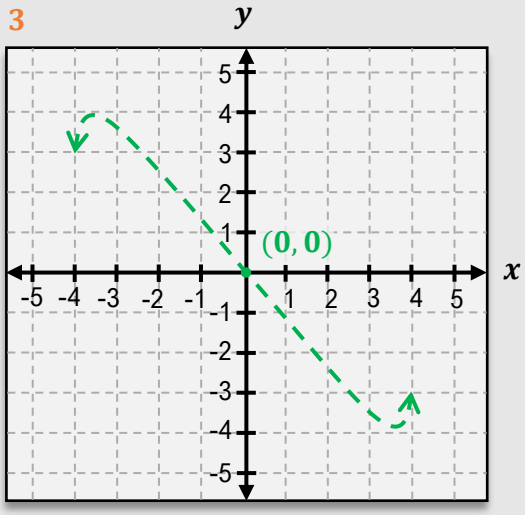
For $f(x-h)$, graph shifts **[RIGHT | LEFT]**

EXAMPLE: Given the function $f(x)$, sketch the transformation: $f(x-2)+3$

$f(x) \rightarrow f(x-h)+k$

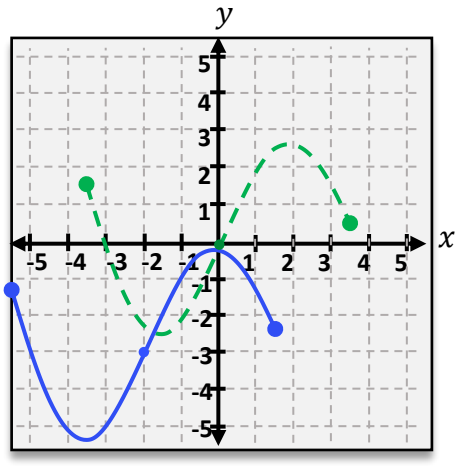
$h =$ _____

$k =$ _____



TOPIC: TRANSFORMATIONS

PRACTICE: The green dotted line in the graph below represents the function $f(x)$. The blue solid line represents the function $g(x)$, which is the function $f(x)$ after it has gone through a shift transformation. Find the equation for $g(x)$.

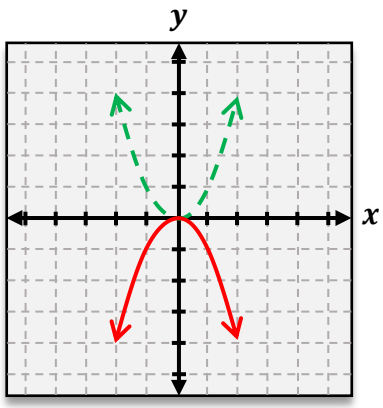
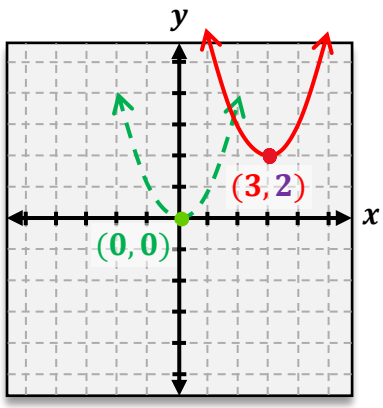
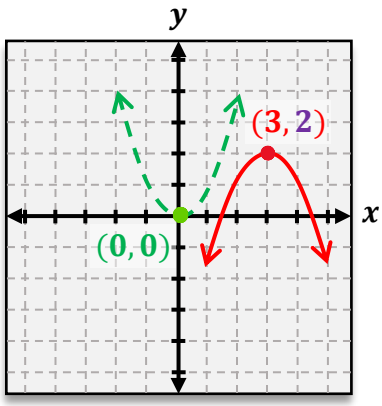


TOPIC: TRANSFORMATIONS

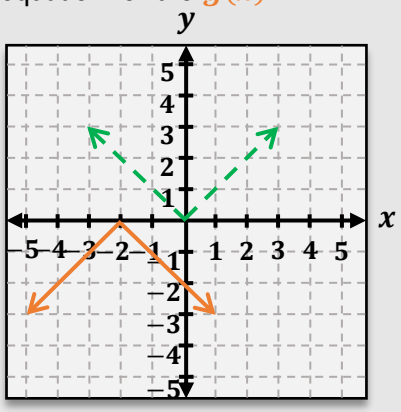
Graphs Of Shifted & Reflected Functions

Transformations of Functions		
Reflection	Shift	Stretch

- It is possible to see a _____ of transformations to a single function.

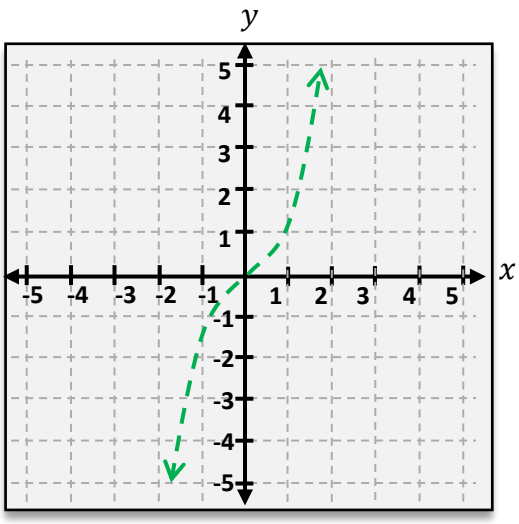
REFLECTION	SHIFTS	REFLECTIONS & SHIFTS
		
$f(x) \rightarrow -f(x)$	$f(x) \rightarrow f(x-3)+2$	$f(x) \rightarrow -f(x-3)+2$

EXAMPLE: If $g(x)$ is a transformation of the function $f(x) = |x|$, write the equation for the $g(x)$.



TOPIC: TRANSFORMATIONS

PROBLEM: If $h(x)$ is a transformation of $f(x) = x^3$ where $f(x)$ is reflected over the ***x-axis***, then shifted down 2 units. (A) write the equation for $h(x)$. (B) Sketch a graph of $h(x)$.



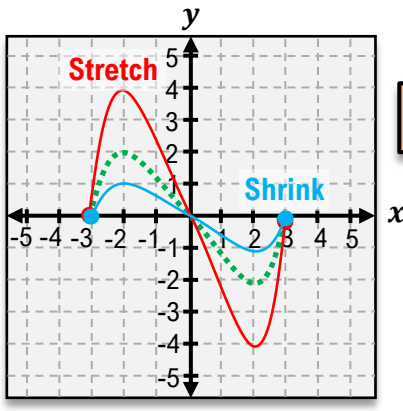
TOPIC: TRANSFORMATIONS

Stretches And Shrinks

Transformations of Functions		
Reflection	Shift	Stretch

- Stretches/Shrinks occur when a _____ (c) is multiplied inside or outside the function.

Vertical Stretch/Compress

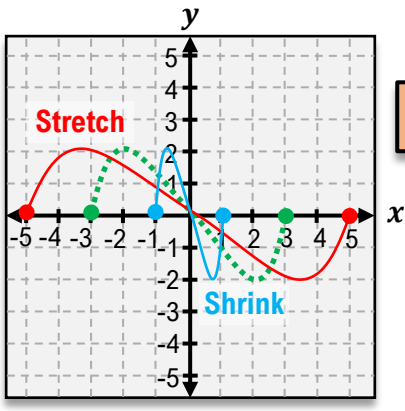


$f(x) \rightarrow c \cdot f(x)$

- When ____< c < ____ the graph vertically
[STRETCHES | SHRINKS]

- When c > ____ the graph vertically
[STRETCHES | SHRINKS]

Horizontal Stretch/Compress



$f(x) \rightarrow f(c \cdot x)$

- When ____< c < ____ the graph horizontally
[STRETCHES | SHRINKS]

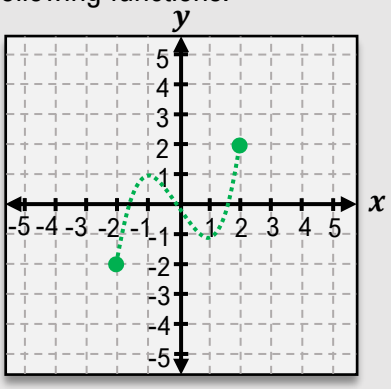
- When c > ____ the graph horizontally
[STRETCHES | SHRINKS]

EXAMPLE: Graphed below is the function $f(x)$. Sketch the graphs of the following functions:

(A) $2 \cdot f(x)$

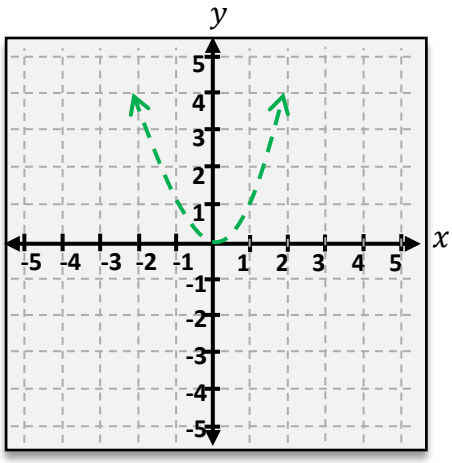
(B) $\frac{1}{2} \cdot f(x)$

(C) $f(\frac{1}{2} \cdot x)$



TOPIC: TRANSFORMATIONS

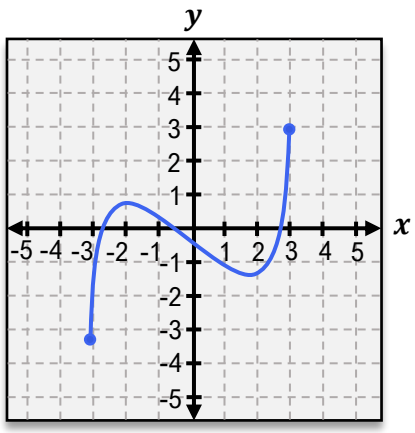
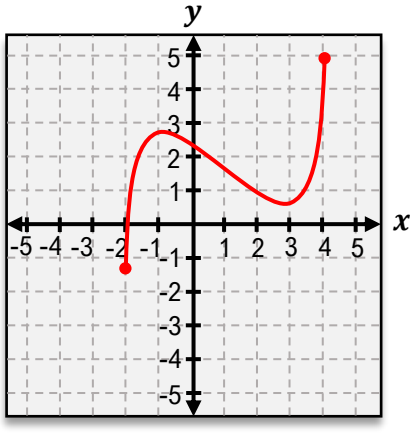
PROBLEM: Given that $f(x) = (c \cdot x)^2$, graph $f(x)$ when $(A)\ c = 2$, $(B)\ c = \frac{1}{2}$



TOPIC: TRANSFORMATIONS

Domain & Range of Transformed Functions

- A transformation can _____ the domain and range of a function.
  - We can find the domain & range of a transformed function by _____ the new graph.

Function: $f(x)$	Function: $f(x-1)+2$
	
Domain → D: $[-3, 3]$ Range → R: $[-3, 3]$	Domain → D: [____, ____] Range → R: [_____, _____]

EXAMPLE: Given the function $f(x) = x^2$, sketch a graph of the function $g(x) = (x-3)^2 + 2$ and determine its domain and range.

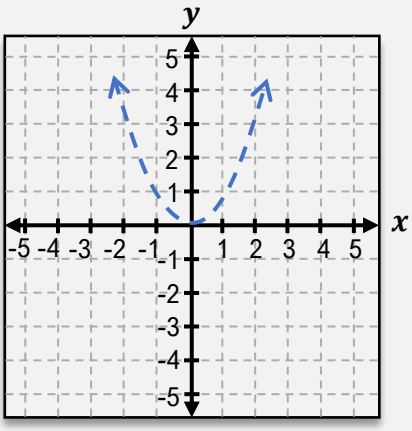
$f(x) \rightarrow f(x-h)+k$

Domain Of $f(x)$: $(-\infty, \infty)$

Range Of $f(x)$: $[0, \infty)$

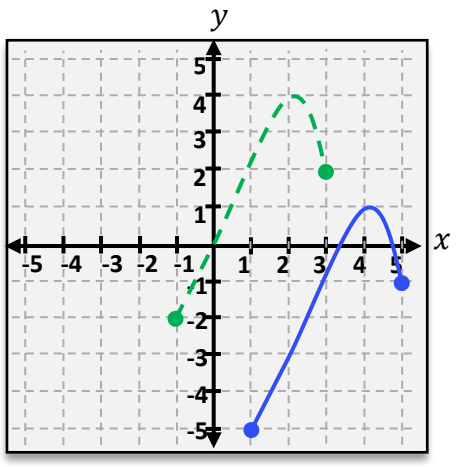
Domain Of $g(x)$:

Range Of $g(x)$:



TOPIC: TRANSFORMATIONS

PRACTICE: The green dotted curve below is a graph of the function $f(x)$. Find the domain and range of $g(x)$ (the blue solid curve), which is a transformation of $f(x)$.



Domain of $f(x) = [-1, 3]$

Range of $f(x) = [-2, 4]$