

TOPIC: PRODUCTS AND QUOTIENTS OF COMPLEX NUMBERS

Products of Complex Numbers in Polar Form

- ◆ Multiply complex numbers (z_1 & z_2) in polar form by _____ the r 's and _____ the θ 's.
- The polar form of complex numbers, $r(\cos \theta + i \sin \theta)$, is often abbreviated $r \cdot \underline{\hspace{1cm}} (\theta)$.

EXAMPLE

Find the product of the complex numbers.

New

Multiply Complex Numbers in Polar Form

$$z_1 \cdot z_2 = r_1 \underline{\hspace{1cm}} r_2 [\cos(\theta_1 \underline{\hspace{1cm}} \theta_2) + i \sin(\theta_1 \underline{\hspace{1cm}} \theta_2)]$$

$$[3(\cos 15^\circ + i \sin 15^\circ)] \cdot [2(\cos 30^\circ + i \sin 30^\circ)]$$

EXAMPLE

Find the product of the complex numbers.

$$\left[4 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)\right] \cdot \left[5 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]$$

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PRACTICE

Given $z_1 = 5(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$ and $z_2 = 3(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4})$, find the product $z_1 \cdot z_2$.

PRACTICE

Given $z_1 = \frac{2}{3}(\cos 25^\circ + i \sin 25^\circ)$ and $z_2 = \frac{5}{2}(\cos 15^\circ + i \sin 15^\circ)$, find the product $z_1 \cdot z_2$.

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Quotients of Complex Numbers in Polar Form

◆ Dividing complex numbers is like multiplication, but with _____ operations.

EXAMPLE Find the quotient.

Recall	Products	New	Quotients
	$[3(\cos 15^\circ + i \sin 15^\circ)] \cdot [2(\cos 30^\circ + i \sin 30^\circ)]$ $3 \cdot 2[\text{cis}(15^\circ + 30^\circ)]$ $6[\text{cis}(45^\circ)]$		$\frac{[6(\cos 45^\circ + i \sin 45^\circ)]}{[3(\cos 15^\circ + i \sin 15^\circ)]}$
	$z_1 \cdot z_2 = r_1 \cdot r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$		$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$
	[MULTIPLY DIVIDE] r 's, and [ADD SUBTRACT] θ 's.		[MULTIPLY DIVIDE] r 's, and [ADD SUBTRACT] θ 's.

EXAMPLE Let $z_1 = 5\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)$ and $z_2 = 4\left(\cos\frac{\pi}{9} + i \sin\frac{\pi}{9}\right)$. Find $\frac{z_1}{z_2}$.

Recall

$$r(\cos\theta + i\sin\theta) = r\text{cis}\theta$$

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PRACTICE

Given $z_1 = \frac{1}{5}(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$ and $z_2 = 5(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5})$, find the quotient $\frac{z_1}{z_2}$.

PRACTICE

Given $z_1 = 12(\cos 30^\circ + i \sin 30^\circ)$ and $z_2 = 3(\cos 50^\circ + i \sin 50^\circ)$, find the quotient $\frac{z_1}{z_2}$.