TOPIC: FINDING LIMITS ALGEBRAICALLY

Finding Limits by Direct Substitution

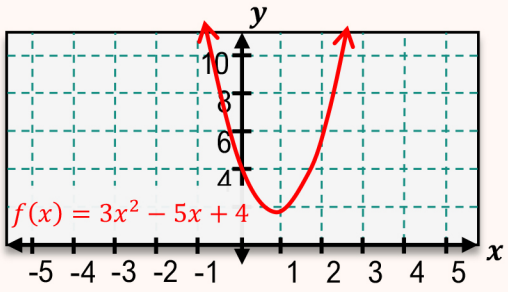
◆ For many functions, like **polynomials** & **basic roots**, the limit is *ALWAYS* the _____ as the function value.

Recall — Evaluating a Function

Evaluate $f(x) = 3x^2 - 5x + 4$ at $x = 2$

$$f(2) = 3(2)^2 - 5(2) + 4$$
$$= 12 - 10 + 4$$
$$= 6$$

New — Finding a Limit



Find $\lim_{x \to 2} f(x)$

$\lim_{x \to 2} f(x) =$

EXAMPLE Find the limit.

(A) $\lim_{x \to 2} 6x^3 + 3x^2 - x + 5$

(B) $\lim_{x \to 0} \sqrt{7x^2 + 4x + 16}$

(C) $\lim_{x \to 0} \frac{x^2 + 3x + 2}{x + 1}$

◆ For **rational** functions, the limit is the same as the function value as long as the denom. ___ 0.

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EXAMPLE Find the limit.

(A) $\lim_{x \to 2} 7$

(B) $\lim_{x \to -1} 2x^2 + 3x$

(C) $\lim_{x \to 3} \sqrt{3x^2 - 2}$

PRACTICE Find the limit.

(A) $\lim_{x \to 0} x^3 + 5x^2 - 7x + 3$

(B) $\lim_{x \to 2} \sqrt{x^2 + 5}$

(C) $\lim_{x \to 3} \frac{x^2 + 2x - 3}{x - 2}$

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Limits of Rational Functions: Denominator = 0

◆ Recall: For **rational** functions, the limit is the same as the function value as long as the denom. $\neq 0$.

- If denom. $= 0$, _____ top & bottom, _____ common factor, then evaluate.

Recall — Denominator $\neq 0$	New — Denominator $= 0$
$\lim_{x\to 0} \frac{x^2+3x+2}{x+1} = \frac{(0)^2+3(0)+2}{0+1}$	$\lim_{x\to -1} \frac{x^2+3x+2}{x+1} =$
$= \frac{2}{1}$	$\lim_{x\to -1}$
$= 2$	$\lim_{x\to -1}$

EXAMPLE Find the limit.

(A) $\lim_{x\to 3} \frac{x^2+2x-15}{x-3}$

(B) $\lim_{x\to -2} \frac{x+2}{x^2-x-6}$

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EXAMPLE Find the limit.

(A)

$$\lim_{x \to 1} \frac{x^3 - 2x^2 + x}{x - 1}$$

(B)

$$\lim_{x \to 2} \frac{x^3 - 2x^2 + x}{x - 1}$$

PRACTICE Find the limit.

(A)

$$\lim_{x \to 0} \frac{3x^2 + 7x}{x}$$

(B)

$$\lim_{x \to 2} \frac{x^2 - 7x + 12}{x - 3}$$

(C)

$$\lim_{x \to -2} \frac{x^2 - 5x - 14}{x + 2}$$

(D)

$$\lim_{x \to -\pi} \frac{\sin x}{x}$$

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Limits of Rational Functions with Radicals

◆ Recall: For **rational** functions, the limit is the same as the function value as long as the denom. $\neq 0$.

- If denom. $= 0$ & function has a $\sqrt{\ }$, multiply top & bottom by _____, cancel common factor, then evaluate.

Recall — Rational Functions

$$\lim_{x\to -1}\frac{x^2+3x+2}{x+1}=\lim_{x\to -1}\frac{(x+1)(x+2)}{x+1}$$

$$=\lim_{x\to -1}(x+2)$$

$$=(-1)+2$$

$$=1$$

New — Rational Functions with Radicals

$$\lim_{x\to 2}\frac{x-2}{\sqrt{x}-\sqrt{2}}$$

Recall

$a+\sqrt{b}\leftrightarrow a-\sqrt{b}$

(Conjugates)

$$\lim_{x\to 2}$$

$$\lim_{x\to 2}$$

EXAMPLE Find the limit.

$$\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}$$

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EXAMPLE Find the limit.

$$\lim_{x\to 0}\frac{\sqrt{4-x}-2}{x}$$

PRACTICE Find the limit.

$(\boldsymbol{A})$

$$\lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{x-3}$$

$(\boldsymbol{B})$

$$\lim_{x\to 0}\frac{\sqrt{x-1}-1}{x}$$

$(\boldsymbol{C})$

$$\lim_{x\to 5}\frac{x-5}{\sqrt{x}-\sqrt{5}}$$