TOPIC: POWERS OF COMPLEX NUMBERS (DEMOIVRE'S THEOREM)

Powers of Complex Numbers in Polar Form (DeMoivre's Theorem)

◆ Find powers of complex numbers in polar form by raising r to the n^{th} ________ and ____________ θ by n.

EXAMPLE Evaluate the expression.

Recall **Products**	New **Powers**
$[3(\cos 15° + i \sin 15°)] \cdot [3(\cos 15° + i \sin 15°)]$	$[3(\cos 15° + i \sin 15°)]^2$
$3 \cdot 3[\text{cis}(15° + 15°)]$	
$9[\text{cis}(30°)]$	
$z_1 \cdot z_2 = r_1 \cdot r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$	$z^n = r \quad [\cos(n \_\_ \theta) + i \sin(n \_\_ \theta)]$ (*DeMoivre's Theorem*)

EXAMPLE Evaluate the following expression.

$$\left[4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]^3$$

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PRACTICE Given $z = 3\text{ cis}\left(\frac{4\pi}{5}\right)$, find the quotient z^5.

EXAMPLE Given the complex numbers $z_1 = 4\text{ cis}(25°)$ and $z_2 = \frac{1}{2}\text{cis}(10°)$, calculate $\left(\frac{z_1}{z_2}\right)^3$

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Finding Roots of Complex Numbers

- Like real #'s, complex roots are just numbers you can raise to the power to get the original number.
  - *Unlike* real #'s, complex numbers have ____________ roots.

Recall: Roots: Real #'s	New: Roots of Complex Numbers	
$2^3 = 8$	$(2 \text{ cis } 15°)^3 = 8 \text{ cis } 45°$	$(8 \text{ cis } 45°)^{\frac{1}{3}}$
$8^{\frac{1}{3}} = 2$	$(r \text{ cis } \theta)^n = r^n \text{ cis}(n\theta)$ *(DeMoivre's Theorem)*	$(r\text{cis}\theta)^{\frac{1}{n}} = r^{\frac{1}{n}}\text{cis}(\theta_k)$ Where, $\theta_k = \frac{1}{-} \cdot (\theta + 2\pi \cdot \_\_\_)$ And $k = 0, 1, 2, \ldots n-1$

EXAMPLE Find the cube roots of $8 \text{ cis } 45°$.

Recall: $r(\cos\theta + i\sin\theta) = r\text{cis}\theta$

$z \ = \ \_\_ \text{ cis}(\_\_\_\_)$

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HOW TO: Find Roots of Complex Numbers

1) Find $r^{\frac{1}{n}}$

For $k = 0, 1, 2, \ldots n-1$

2) Set up $z_k = r^{\frac{1}{n}} \text{ cis}(\theta_k)$

3) Find $\theta_k = \frac{1}{n} \cdot (\theta + 2\pi^* k)$

*or 360°

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EXAMPLE If $z = 1024\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$, calculate $\sqrt[5]{z}$.