Determinants of 2 × 2 Matrices

- ◆ A **Determinant** is simply a number calculated from a matrix. Later, we'll use it to solve systems of equations.
 - ▶ In a 2 \times 2 matrix, calculate this by subtracting the products of the _____.

Determinant of 2x2 Matrix

$$\det \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (_\cdot_) - (_\cdot_)$$

$$A = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$$

$$\det(A) \text{ or } |A| = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} =$$

EXAMPLE

Evaluate the determinant of the matrix.

$$A = \begin{bmatrix} 8 & 4 \\ 5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & -7 \\ 1 & -2 \end{bmatrix}$$

PRACTICE Evaluate the determinant of the matrix.

$$\begin{bmatrix} 4 & \frac{9}{2} \\ \frac{2}{3} & 9 \end{bmatrix}$$

Recall
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \cdot d) - (b \cdot c)$$

(2x2 Determinant)

TOPIC: MATRIX DETERMINANTS & CRAMER'S RULE Cramer's Rule – Systems of 2 Equations w/ 2 Unknowns

- ◆ Cramer's Rule is a formula that *directly* gives the solution to a system of linear equations.
 - Given 2 equations with 2 unknown variables, Cramer's Rule uses determinants of ____ × ___ matrices.

EXAMPLE

Solve the system of equations using Cramer's Rule.

$$2x + y = 5$$
$$-4x + 6y = -2$$

Given
$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \\ a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \\ a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$
If
$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$$
, there is _____ solution

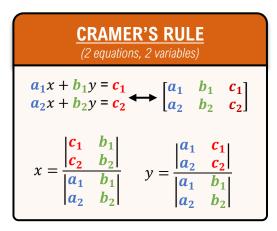
Determinant of 2x2 Matrix
$$det \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \cdot d) - (b \cdot c)$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \cdot d) - (b \cdot c)$$

(2x2 Determinant)

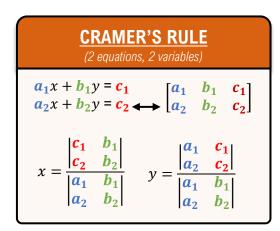
PRACTICE Write each equation in standard form and use Cramer's Rule to solve the system.

$$y = -3x + 4$$
$$-2x = 7y - 9$$



PRACTICE Write each equation in standard form and use Cramer's Rule to solve the system.

$$y - 9x = -3$$
$$-3x = 4y - 1$$



TOPIC: MATRIX DETERMINANTS & CRAMER'S RULE Determinants of 3 × 3 Matrices

ullet Determinants of 3 imes 3 matrices are more complicated and involve calculating multiple 2 imes 2 determinants.

Recall 2 × 2 Determinant	New 3 × 3 Determinant
$det \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \cdot d) - (b \cdot c)$	$det \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} - \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$ signs alternate

EXAMPLE

Evaluate the determinant of the matrix.

$$\begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & -3 \\ -1 & -4 & 6 \end{bmatrix}$$

PRACTICE Evaluate the determinant of the matrix.

$$\begin{bmatrix} 4 & 1 & 2 \\ -5 & 2 & -3 \\ 1 & -4 & 6 \end{bmatrix}$$

Recall
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

(3x3 Determinant)

TOPIC: MATRIX DETERMINANTS & CRAMER'S RULE Cramer's Rule – Systems of 3 Equations w/ 3 Unknowns

- ◆ Remember: Cramer's Rule *directly* gives the solution to a system of linear equations.
 - ▶ Given 3 equations w/ 3 unknown variables, Cramer's Rule uses determinants of ___ × ___ matrices.

EXAMPLE

Solve the system of equations using Cramer's Rule.

$$-5x - y + 4z = 4$$
$$3y + 6z = 21$$
$$x + y + z = 6$$

Cramer's Rule
(3 equations, 3 variables)

$$a_1x + b_1y + c_1z = d_1$$
Given
$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$
If
$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$
, then
$$x = \frac{D_x}{D} \rightarrow D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$
 (replace x coeffs with "constants")
$$y = \frac{D_y}{D} \rightarrow D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$
 (replace y coeffs with "constants")
$$z = \frac{D_z}{D} \rightarrow D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$
 (replace y coeffs with "constants")

Recall Determinant of 3 × 3 Matrix $det \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$ $a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

PRACTICE Solve the system of equations using Cramer's Rule.

$$4x + 2y + 3z = 6$$
$$x + y + z = 3$$
$$5x + y + 2z = 5$$

Cramer's Rule
(3 equations, 3 variables)

$$a_{1}x + b_{1}y + c_{1}z = d_{1}$$

$$a_{2}x + b_{2}y + c_{2}z = d_{2}$$

$$a_{3}x + b_{3}y + c_{3}z = d_{3}$$

$$x = \frac{D_{x}}{D} \rightarrow \begin{bmatrix} d_{1} & b_{1} & c_{1} \\ d_{2} & b_{2} & c_{2} \\ d_{3} & b_{3} & c_{3} \end{bmatrix}$$

$$y = \frac{D_{y}}{D} \rightarrow \begin{bmatrix} a_{1} & d_{1} & c_{1} \\ d_{2} & b_{2} & c_{2} \\ d_{3} & d_{3} & c_{3} \end{bmatrix}$$

$$z = \frac{D_{z}}{D} \rightarrow \begin{bmatrix} a_{1} & d_{1} & c_{1} \\ a_{2} & d_{2} & c_{2} \\ a_{3} & d_{3} & c_{3} \end{bmatrix}$$

$$z = \frac{D_{z}}{D} \rightarrow \begin{bmatrix} a_{1} & b_{1} & d_{1} \\ a_{2} & b_{2} & d_{2} \\ a_{3} & b_{3} & d_{3} \end{bmatrix}$$