TOPIC: MATRIX DETERMINANTS & CRAMER'S RULE
Determinants of 2 × 2 Matrices

◆ A **Determinant** is simply a number calculated from a matrix. Later, we'll use it to solve systems of equations.

- In a 2×2 matrix, calculate this by subtracting the products of the ______________.

Determinant of 2x2 Matrix

$$det\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (\_ \cdot \_) - (\_ \cdot \_)$$

$$A = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$$

$$\det(A) \text{ or } |A| = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} =$$

EXAMPLE Evaluate the determinant of the matrix.

(A)

$$A = \begin{bmatrix} 8 & 4 \\ 5 & 0 \end{bmatrix}$$

(B)

$$B = \begin{bmatrix} -3 & -7 \\ 1 & -2 \end{bmatrix}$$

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PRACTICE Evaluate the determinant of the matrix.

$$\begin{bmatrix} 4 & \frac{9}{2} \\ \frac{2}{3} & 9 \end{bmatrix}$$

Recall $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \cdot d) - (b \cdot c)$

(2x2 *Determinant*)

TOPIC: MATRIX DETERMINANTS & CRAMER'S RULE
Cramer's Rule – Systems of 2 Equations w/ 2 Unknowns

- **Cramer's Rule** is a formula that ***directly*** gives the solution to a system of linear equations.
  - Given 2 equations with 2 unknown variables, Cramer's Rule uses determinants of ___ × ___ matrices.

EXAMPLE Solve the system of equations using Cramer's Rule.

$$2x + y = 5$$
$$-4x + 6y = -2$$

Cramer's Rule
(2 equations, 2 variables)

Given

$$a_1x + b_1y = c_1 \qquad \longleftrightarrow \qquad \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$$
$$a_2x + b_2y = c_2$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

If $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$, there is ____ solution

Recall **Determinant of 2x2 Matrix**

$$det\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \cdot d) - (b \cdot c)$$

Recall

$$det\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \cdot d) - (b \cdot c)$$

(2x2 *Determinant*)

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PRACTICE Write each equation in standard form and use Cramer's Rule to solve the system.

$$y = -3x + 4$$
$$-2x = 7y - 9$$

CRAMER'S RULE

(2 equations, 2 variables)

$$a_1x + b_1y = c_1 \qquad a_2x + b_2y = c_2 \longleftrightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

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PRACTICE Write each equation in standard form and use Cramer's Rule to solve the system.

$$y - 9x = -3$$
$$-3x = 4y - 1$$

CRAMER'S RULE

(2 equations, 2 variables)

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2 \longleftrightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

TOPIC: MATRIX DETERMINANTS & CRAMER'S RULE
Determinants of 3 × 3 Matrices

◆ Determinants of 3 × 3 matrices are more complicated and involve calculating multiple 2 × 2 determinants.

Recall **2 × 2 Determinant**	New **3 × 3 Determinant**
$det\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \cdot d) - (b \cdot c)$	$det\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$ $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} - \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $= a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$ signs alternate

EXAMPLE Evaluate the determinant of the matrix.

$$\begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & -3 \\ -1 & -4 & 6 \end{bmatrix}$$

TOPIC: MATRIX DETERMINANTS & CRAMER'S RULE

PRACTICE Evaluate the determinant of the matrix.

$$\begin{bmatrix} 4 & 1 & 2 \\ -5 & 2 & -3 \\ 1 & -4 & 6 \end{bmatrix}$$

Recall

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$$

$$a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

(3x3 *Determinant*)

TOPIC: MATRIX DETERMINANTS & CRAMER'S RULE
Cramer's Rule – Systems of 3 Equations w/ 3 Unknowns

◆ Remember: **Cramer's Rule** *directly* gives the solution to a system of linear equations.

- Given 3 equations w/ 3 unknown variables, Cramer's Rule uses determinants of __ × __ matrices.

EXAMPLE Solve the system of equations using Cramer's Rule.

$$-5x - y + 4z = 4$$
$$3y + 6z = 21$$
$$x + y + z = 6$$

Cramer's Rule
(3 equations, 3 variables)

Given
$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$, then

$x = \frac{D_x}{D} \longrightarrow D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$ *(replace x coeff's with "constants")*

$y = \frac{D_y}{D} \longrightarrow D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ *(replace y coeff's with "constants")*

$z = \frac{D_z}{D} \longrightarrow D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$ *(replace y coeff's with "constants")*

Recall: Determinant of 3 × 3 Matrix

$$det \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$$

$$a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

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PRACTICE Solve the system of equations using Cramer's Rule.

$$4x + 2y + 3z = 6$$
$$x + y + z = 3$$
$$5x + y + 2z = 5$$

Cramer's Rule

(3 equations, 3 variables)

$$\begin{matrix} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{matrix} \longleftrightarrow \left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

$$x = \frac{D_x}{D} \rightarrow \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}$$

$$y = \frac{D_y}{D} \rightarrow \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}$$

$$z = \frac{D_z}{D} \rightarrow \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$