TOPIC: UNDERSTANDING POLYNOMIAL FUNCTIONS

Intro to Polynomial Functions

- You will need to know how recognize polynomial functions & their graphs.
    - Recall: Polynomials have *only* positive whole number exponents (*no negatives, no fractions*)
    - Standard form: Like terms combined & in descending order of power $f(x) \;= a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

Degree n (highest exponent)

$$\_\_\_\_\_ = 6x^3 + 3x^2 + 5x + 4$$

Leading Coefficient　　Coefficients　　Constant

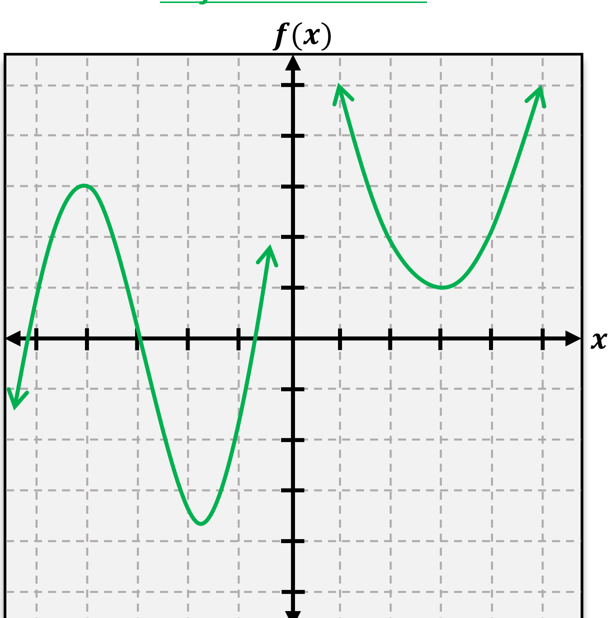
EXAMPLE: Determine if each function is a polynomial function. If so, put in standard form. State degree & leading coeff.

(A)	(B)	(C)
$f(x) = -x^2 + 5x^3 - 6x + 4$	$f(x) = 2x^{\frac{1}{2}} + 3$	$f(x) = -\frac{2}{3}x^4 + 1 + 3$
Polynomial function? ☐	Polynomial function? ☐	Polynomial function? ☐
Degree: _______	Degree: _______	Degree: _______
Leading Coefficient: _______	Leading Coefficient: _______	Leading Coefficient: _______

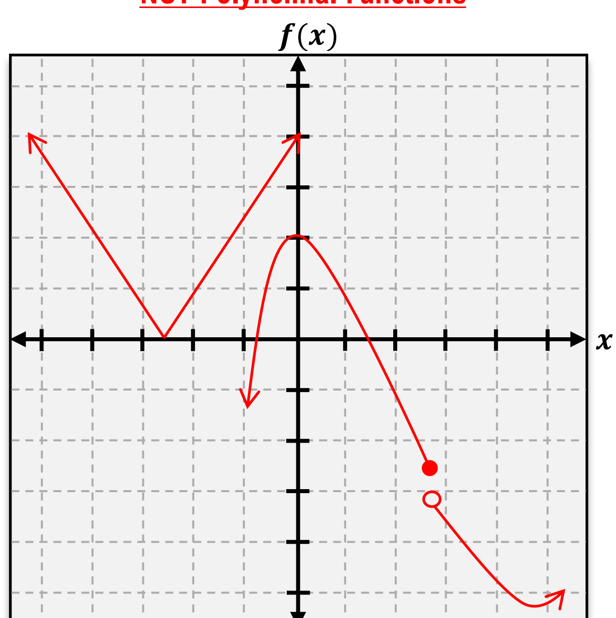
Graphs of Polynomial Functions

- Graphs of polynomial functions are _____________ and ________________ (*no corners, no breaks*)

Polynomial Functions



NOT Polynomial Functions



- Domain: *always* ___________

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PRACTICE: Determine if the given function is a polynomial function. If so, write in standard form, then state the degree and leading coefficient.

$$f(x) = 4x^3 + \frac{1}{2}x^{-1} - 2x + 1$$

Polynomial function? ☐
Degree: _______
Leading Coefficient: _______

PRACTICE: Determine if the given function is a polynomial function. If so, write in standard form, then state the degree and leading coefficient.

$$f(x) = 2 + x$$

Polynomial function? ☐
Degree: _______
Leading Coefficient: _______

PRACTICE: Determine if the given function is a polynomial function. If so, write in standard form, then state the degree and leading coefficient.

$$f(x) = 3x^2 + 5x + 2$$

Polynomial function? ☐
Degree: _______
Leading Coefficient: _______

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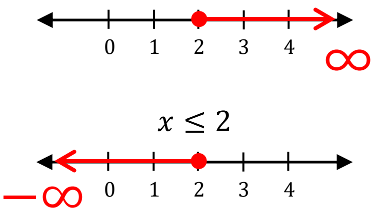
End Behavior

- Just as the graph of an inequality may go to $+\infty$ **or** $-\infty$, the graph of a polynomial function will *always* do this.
  - **End Behavior:** what the graph of $\boldsymbol{f(x)}$ does far to the *left* (________) and far to the *right* (________)
    "x approaches $-\infty$" "x approaches ∞"

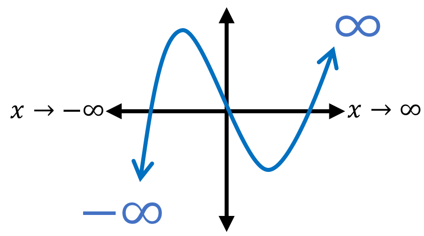
Graphing Inequalities

$x \geq 2$

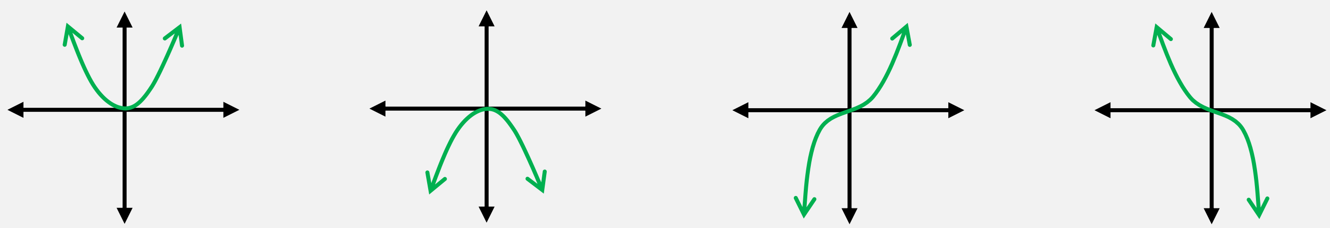
$x \leq 2$



Graphing Functions



- The behavior in the *middle* of the graph will look different depending on the function.



a_n is [+ | −]	a_n is [+ | −]	a_n is [+ | −]	a_n is [+ | −]
n is [EVEN | ODD]	n is [EVEN | ODD]	n is [EVEN | ODD]	n is [EVEN | ODD]

- To determine the end behavior of a polynomial, look at the __________ term in standard form.
  - If Leading Coefficient (a_n) is: **positive**: right side __________ ($f(x) \rightarrow$______)
    negative: right side __________ ($f(x) \rightarrow$______)

    Degree (n) is: **even**: ends are _______________
    odd: ends are _______________

Degree

$3x^5 + \cdots$

Leading Coefficient

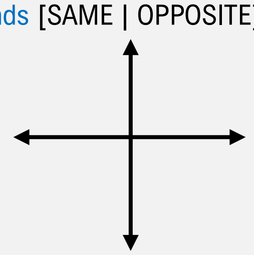
EXAMPLE: Determine the end behavior of each polynomial function, then sketch.

(A)

$$f(x) = -4x^6 + x^3 + 2$$

Right [RISES | FALLS]

Ends [SAME | OPPOSITE]

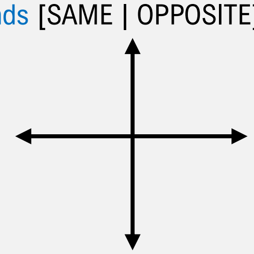


(B)

$$f(x) = 2x^3 + x$$

Right [RISES | FALLS]

Ends [SAME | OPPOSITE]



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PRACTICE: Determine the end behavior of the given polynomial function.

$f(x) = x^2 + 4x + x + 7x^3$

Right side [RISES | FALLS]

Ends are [SAME | OPPOSITE]

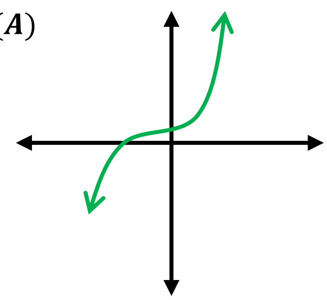
PRACTICE: Match the given polynomial function to its graph based on end behavior.

$f(x) = -2x^3 + x^2 + 1$

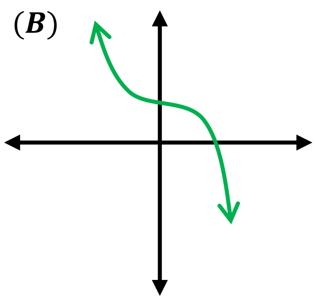
Right side [RISES | FALLS]

Ends are [SAME | OPPOSITE]

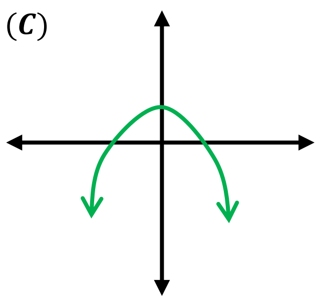
$(\boldsymbol{A})$



$(\boldsymbol{B})$



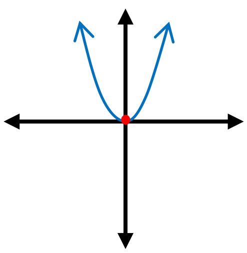
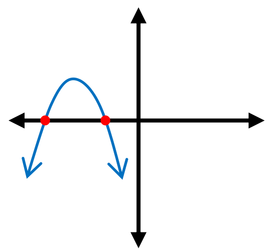
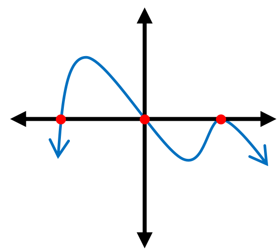
$(\boldsymbol{C})$



TOPIC: UNDERSTANDING POLYNOMIAL FUNCTIONS
Finding Zeros by Factoring & Multiplicity of Zeros

- There may be multiple **x-intercepts** of a polynomial function, called the *zeros, roots* or *solutions* of $f(x) = 0$.
  - Find zeros by factoring & setting each factor $= 0$.
  - **Multiplicity** of a zero: __________ of times a factor occurs.

$(x-1)^2$
$x - 1 = 0 \rightarrow$ $x = 1$ has multiplicity ___
$x = 1$

EXAMPLE**:** Find the zeros of the given polynomial function and give the multiplicity of each. State whether the graph crosses or touches the x-axis at each zero.

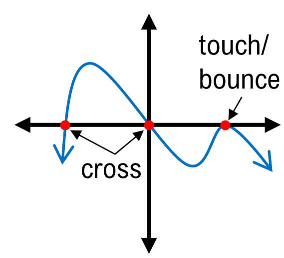
$$f(x) = 2x(x-3)^2(x+4)^3$$

$x =$ __________ Multiplicity: ____ [TOUCH | CROSS]

$x =$ __________ Multiplicity: ____ [TOUCH | CROSS]

$x =$ __________ Multiplicity: ____ [TOUCH | CROSS]

- Multiplicity determines what the graph does at that zero: If even: graph __________ x-axis
  odd: graph __________ x-axis



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PRACTICE: Find the zeros of the given polynomial function and give the multiplicity of each. State whether the graph crosses or touches the x-axis at each zero.

$$f(x) = 2x^4 - 12x^3 + 18x^2$$

$x =$ __________ $x =$ __________

Multiplicity: ____ Multiplicity: ____

[TOUCH | CROSS] [TOUCH | CROSS]

PRACTICE: Find the zeros of the given polynomial function and give the multiplicity of each. State whether the graph crosses or touches the x-axis at each zero.

$$f(x) = x^2\,(x-1)^3(2x+6)$$

$x =$ __________ $x =$ __________ $x =$ __________

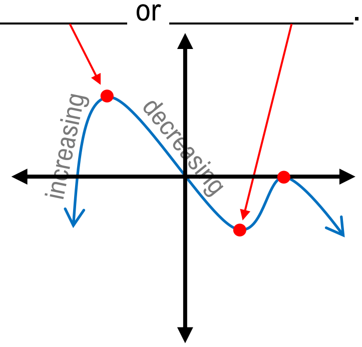
Multiplicity: ____ Multiplicity: ____ Multiplicity: ____

[TOUCH | CROSS] [TOUCH | CROSS] [TOUCH | CROSS]

TOPIC: UNDERSTANDING POLYNOMIAL FUNCTIONS
Turning Points

● Graphs of polynomial functions may have multiple **turning points**: points where the graph changes ____________.

- The *maximum* number of turning points is $n - 1$, where n is the _________ of the polynomial.
- Each turning point is either a local ___________ or _____________.



EXAMPLE: Determine the maximum number of turning points for each polynomial function.

(A)	(B)	(C)
$f(x) = 6x^4 + 2x$	$f(x) = x^2 - 1$	$f(x) = -x^2 + 5x^3 - 6x$
Max. turning points: _______	Max. turning points: _______	Max. turning points: _______

● When graphing, we'll be able to use turning points to check that we graphed correctly.

PRACTICE: Determine the maximum number of turning points for the given polynomial function.

$$f(x) = 6x^4 + 2x$$

Max. turning points: _______

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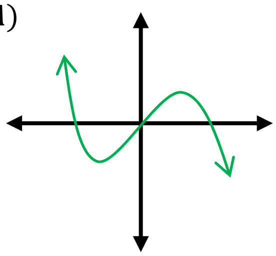
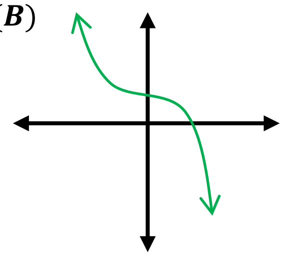
PRACTICE**:** Based *ONLY* on the maximum number of turning points, which of the following graphs could NOT be the graph of the given function?

$$f(x) = x^3 + 1$$

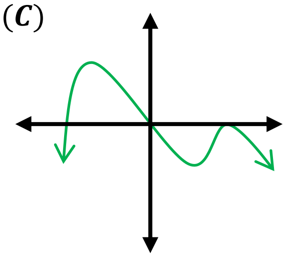
$(\boldsymbol{A})$



$(\boldsymbol{B})$



$(\boldsymbol{C})$



PRACTICE**:** The given term represents the leading term of some polynomial function. Determine the end behavior and the maximum number of turning points.

$4x^5$

Right side [RISES | FALLS]

Ends are [SAME | OPPOSITE]

Max. turning points: ________