TOPIC: BINOMIAL DISTRIBUTION

The Binomial Experiment

◆ A common example of a Disc. Rand. Var. (X) is the outcomes of a binomial experiment:

► "Binomial" = events with ___ outcomes: success & failure (e.g. heads/tails in coin flip). x = # of ____________

► "Experiment" = Series of n independent _______; each trial has P(success)= __ , P(failure) $= q =$ _______

Recall
$P(A') = 1 - P(A)$

EXAMPLE Determine whether the following experiments are binomial experiments. If they are, identify the values of $n, p, q,$ and x.

(A) You flip a coin 4 times. You count the number of times the coin lands on heads, which is 3 times.

☐ Only 2 outcomes?

☐ Fixed # of trials?

☐ Independent trials?

☐ Equal P(success) per trial?

(B) You pull 4 marbles out of a bag of red & blue marbles *without* replacing them. The chance of getting a blue marble is 60%. You get one 1 blue marble.

☐ Only 2 outcomes?

☐ Fixed # of trials?

☐ Independent trials?

☐ Equal P(success) per trial?

PRACTICE A basketball player normally has a 70% chance of making a free throw. The player shoots until finally making a basket, where x is the number of shots they take. Is this a binomial experiment?

(A) Yes

(B) No, there are not 2 outcomes

(C) No, there are not a fixed # of trials

(D) No, the trials are not independent

(E) No, the probability of success is not the same for each trial

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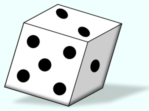
EXAMPLE

You roll a six-sided die 10 times and record the number landed on for each roll as the random variable. Is this a binomial experiment?



(A) Yes
(B) No, there are not 2 outcomes
(C) No, there are not a fixed # of trials
(D) No, the trials are not independent
(E) No, the probability of success is not the same for each trial

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Using the above example, now you roll another 10 times and choose the random variable as the number of times you roll exactly a 2. Is this a binomial experiment?

(A) Yes
(B) No, there are not 2 outcomes
(C) No, there are not a fixed # of trials
(D) No, the trials are not independent
(E) No, the probability of success is not the same for each trial

EXAMPLE

A pharmaceutical company claims the probability of gaining immunity from a new vaccine is 80%. The vaccine was given to 100 patients, and 83 gained immunity. Is this a binomial experiment? If so, identify values of $n, p, q,$ and x.

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Probabilities of Exact Successes in a Binomial Distribution

◆ You will often have to calculate the **probability** of getting $\boldsymbol{X}$ successes $P(X = x)$ in a binomial experiment.

EXAMPLE

You put red & blue marbles in a bag. You take out one marble randomly and ***replace*** it, repeating this for a total of 4 times. If the probability of getting a red marble is 80%, what is the probability of getting exactly 3 red marbles?

New **Probability Distribution**

1 Trial = __________ Success = _____ Failure = ___________

$n =$ _____ $p =$ ______ $q =$ _______________

$x =$ _______:

$P(X = 3) = P(\_) \cdot P(\_) \cdot P(\_) \cdot P(\_)$ · (# ways to _______ 3 R's)

$$P(X = x) = P(x) = (\quad) \cdot \_\_^{\_} \cdot \_\_^{\_\_\_\_\_\_}$$

(Binomial Probability Formula)

HOW TO: Find Prob. for Var with Binomial Distribution

1) Define *trial*, *success*, & *failure*

2) Identify $n, p, q, \& \ x$

$n =$ # trials

$p = P(success)$

$x =$ "desired" # of successes

3) Use formula to evaluate $P(X = x)$

Probability of multiple events:

Recall

$P(A \cap B) = P(A) \cdot P(B)$

*A and B are independent

(Multiplication Rule)

ways to arrange k out of n objects:

Recall

$${}_nC_k = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

(Combination)

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PRACTICE

You take a 6-question quiz with True/False questions. What is the probability of getting all 6 questions correct by simply guessing?

Recall

$$P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

(Binomial Probability)

Recall

$${}_nC_k = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

(Combination)

PRACTICE

National surveys indicate that 36% of people have been in a car accident in the last 5 years. If you randomly sample 10 people, how likely is it that exactly 4 have had an accident in the last 5 years?

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Mean and Standard Deviation of Binomial Distribution

◆ Instead of using summations, find μ_x & σ_x of any binomial probability distribution using just n & P.

EXAMPLE

A company offers a new product, and 60% of customers are expected to purchase it after a demonstration. If you randomly select 10 customers, how many would you expect to purchase the product? What is the standard deviation and variance of this distribution?

Recall — Exp. Value & Std. Dev. for any DRV	New — Exp. Value & Std. Dev. of Binomial Dist.
$\mu = E(X) = \sum x_i \cdot P(x_i)$ $\sigma = \sqrt{\sum [X^2 \cdot P(X)] - \mu^2}$ $\mu = 1 \cdot P(X=1) + 2 \cdot P(X=2) + \ldots + 6 \cdot P(X=6)$	$\mu_x =$ ______ $\sigma_x =$ ______ $n =$ ____ $p =$ ____

PRACTICE

Based on historical weather data in a certain city, about 62% of the days are cloudy. Find the mean, standard deviation, and variance for the number of cloudy days in a 30-day month.

Recall

$\mu_x = np$

$\sigma_x = \sqrt{npq}$

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Multiple Probabilities in Binomial Distributions

◆ When you see keywords such as "**between**" or "**at least**", simply _______ probabilities.

Common Terms for Finding Probabilities					
"exactly $\boldsymbol{a}$"	"less than $\boldsymbol{b}$"	"at most $\boldsymbol{b}$"	"between $\boldsymbol{a}$ & $\boldsymbol{b}$"	"at least $\boldsymbol{a}$"	"more than $\boldsymbol{a}$"
$P(X = a)$	$P(0) +$ $\dots P(X = b - 1)$	$P(0) +$ $\dots P(X = b)$	$P(X = a) +$ $\dots P(X = b)$	$P(X = a) +$ $\dots P(X = n)$	$P(X = a + 1) +$ $\dots P(X = n)$

EXAMPLE

A marketing company wants to analyze how many customers will open a promotional email, so they send it to 10 customers. Based on past campaigns, 40% of people typically open the email.

$(\boldsymbol{A})$ What is the probability that between 0-2 customers open the email?

Recall

$$P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

(Binomial Distribution)

$(\boldsymbol{B})$ What is the probability that at least 3 open the email?

Recall

$$P(A') = 1 - P(A)$$

(Prob. of Complement)

◆ Sometimes it may be easier to *subtract* the probability of the complement.

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PRACTICE

A gardener plants 8 seeds, and each as a 65% probability of germinating successfully. Find the probability that less than 4 seeds germinate successfully.

Recall

$$P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

(Binomial Distribution)

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Finding Binomial Probabilities Using a TI-84

- For *exact* probabilities, use **`binompdf`**. For *cumulative* prob's (<, >, "at least", "between", etc.) use **`binomcdf`**.
  - The **`binomcdf`** gives the probability of values ______ **`x value`**.

EXAMPLE

A manufacturing company produces light bulbs, with a defect rate of 5%. The company produces 1,000 light bulbs in a day. Find each probability.

$n =$ _________ , $p =$ _________

$(\boldsymbol{A})$ Exactly 50 light bulbs are defective.

____

$(\boldsymbol{B})$ At most 50 light bulbs are defective.

____

$(\boldsymbol{C})$ Less than 50 light bulbs are defective.

HOW TO: Find Binomial Probabilities on TI-84

1) 2ND VARS **DISTR**

2) V If Exact: **`A:binompdf(`**

If Cumulative: **`B:binomcdf(`**

3) **`trials:`**

`p:`

`x value:`

PRACTICE

A biologist is monitoring a large bird sanctuary where a particular bird species is known to have a 70% success rate for each nesting attempt (at least one chick fledges from the nest). This season, she observes 500 independent nesting attempts across the sanctuary.

$(\boldsymbol{A})$ What is the probability that exactly 450 nesting attempts are successful?

____

$(\boldsymbol{B})$ What is the probability that less than 330 attempts are successful?

____

$(\boldsymbol{C})$ What is the probability that at least 330 nesting attempts are successful?

____

$(\boldsymbol{D})$ What is the probability that 330 – 370 nesting attempts are successful?

HOW TO: Find Binomial Probabilities on TI-84

1) 2ND VARS **DISTR**

2) V If Exact: **`A:binompdf(`**

If Cumulative: **`B:binomcdf(`**

3) **`trials:`**

`p:`

`x value:`

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EXAMPLE

A company tracks the quality of its customer service calls. Based on historical data, the company estimates that 90% of all customer service calls result in a satisfied customer. On a randomly selected day, 20 customer service calls are made.

(***A***) What is the probability of 15 or fewer customers out of the 20 being satisfied?

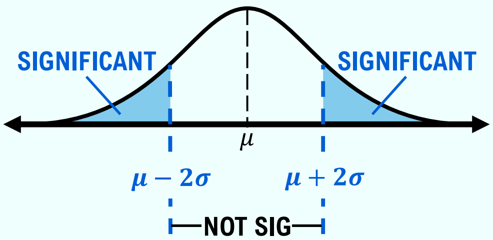
Recall

$$P(X = x) = (n, k) \cdot p^x \cdot q^{n-x}$$

$$\mu_x = np, \qquad \sigma_x = \sqrt{npq}$$

(Binomial Distribution)

(***B***) Use the range rule of thumb to determine whether observing 15 or fewer satisfied customers is statistically unusual. Should the manager be concerned about a possible decline in service quality?



EXAMPLE

A medical researcher is studying the effectiveness of a new vaccine. In clinical trials, the vaccine is known to be 90% effective in preventing infection when administered properly. The researcher selects a random sample of 10 patients who received the vaccine and checks whether they became infected after exposure.

(***A***) Find the probability that the vaccine was effective for 7 or fewer patients.

Recall

$$P(X = x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

(Binomial Distribution)

Recall

$$P(A') = 1 - P(A)$$

(***B***) If the researcher finds the vaccine was effective for 7 or fewer patients, is this result "statistically significant"? Use levels of significance: 10%, 5%, & 1%. *(Hint: compare prob. to each level of significance)*

$P(X \leq 7) =$ ________ **[< | >]** 1%, so **[SIG. | NOT SIG.]**

$P(X \leq 7) =$ ________ **[< | >]** 5%, so **[SIG. | NOT SIG.]**

$P(X \leq 7) =$ ________ **[< | >]** 10%, so **[SIG. | NOT SIG.]**