

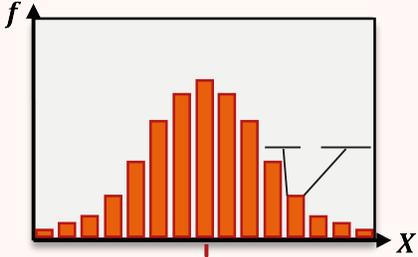
TOPIC: SAMPLING DISTRIBUTION OF SAMPLE PROPORTION

Using the Normal Distribution to Approximate Binomial Probabilities

- ◆ Recall: In a binomial dist., n trials w/ 2 outcomes (S or F); $x = \#$ of successes, $p = 1 - q = P(S) = \text{constant}$.
 - ▶ Instead, for $np \geq \underline{\hspace{1cm}}$ & $nq \geq \underline{\hspace{1cm}}$ use a z -score to find probabilities instead of the binomial formula.

EXAMPLE

The probability of someone voting for a particular candidate in a two-person election is 56%. Use a normal distribution to approximate the probability that more than 60 of a sample of 100 people vote for the candidate.

Recall Prob. w/ Binomial Formula	New Prob. w/ Normal Approximation
<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$ $P(a < X < b) = \sum_{x=a}^b P(X = x)$ </div> <p> $P(X > 60) = P(X = 60) + \dots + P(X = 100)$ $= \binom{100}{60} 0.45^{60} 0.55^{40} + \dots$ $+ \binom{100}{100} 0.45^{100} 0.55^0$ $= ?$ </p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Recall</p> $\mu_x = np, \sigma_x = \sqrt{npq}$ </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> $z = \frac{x - \mu_x}{\sigma_x} = \frac{x - \mu_x}{\sqrt{npq}}$ </div> <div style="text-align: right;">  </div> <ol style="list-style-type: none"> 1) Verify $np \geq 5$ <input type="checkbox"/> and $nq \geq 5$ <input type="checkbox"/> 2) Determine if you need to add or subtract 0.5 to x 3) Find z-score using equation above 4) Determine probability using a table or calculator

- ◆ When approximating binomial distribution as normal, make a **continuity correction**:

Binomial	Normal
Exactly c	$P(c - 0.5 < x < c + 0.5)$
Between c & d	$P(c - 0.5 < x < d + 0.5)$
At most c	$P(x < c + 0.5)$
Fewer than c	$P(x < c - 0.5)$
At least c	$P(x > c - 0.5)$
More than c	$P(x > c + 0.5)$

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PRACTICE

The probability of someone voting for a particular candidate in a two-person election is 45%. Use a normal distribution to approximate the probability that between 62 and 70 people out of a sample of 100 vote for the candidate.

PRACTICE

A previous study found that 80% of people preferred drinking Pepsi over Coca Cola. Use a normal distribution to approximate the probability that a random sample of 100 people reveals at least 60 people preferring Pepsi.

PRACTICE

A previous study found that 80% of people preferred drinking Pepsi over Coca Cola. Use a normal distribution to approximate the probability that, from this same random sample of 100 people, that between 10 and 11 people prefer Coca Cola.

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EXAMPLE

In 2023 about 35 million cars were recalled, out of approximately 94 million cars produced. A used car dealer has 76 cars that were produced in 2023. Use a normal distribution to approximate the probability that more than half of these cars were recalled.

TOPIC: SAMPLING DISTRIBUTION OF SAMPLE PROPORTION

Sampling Distribution of Sample Proportion

- ◆ Recall: For binomial variables (X) when $np \geq 5$ & $nq \geq 5$, X has an approximately normal distribution.
 - ▶ For a binomial experiment, the **Sample Proportion** (\hat{p}) is the actual # of successes out of the # of _____.
 - ▶ Take many random samples to get a **sample distribution** of \hat{p} .

EXAMPLE

You and several classmates take turns flipping a coin 10 times each. The distribution of heads (successes) is shown below. Find the mean and standard deviation of the sampling distribution for \hat{p} .

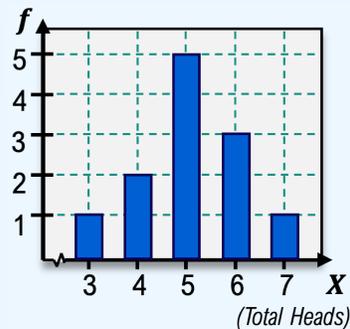
Recall Distribution of Binomial Variable X

$$\mu_X = np, \sigma_X = \sqrt{npq}$$

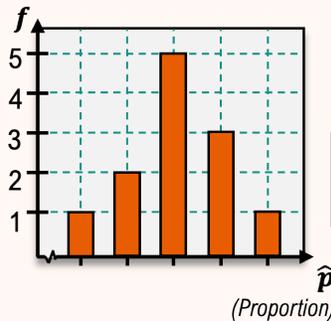
$$n = 10, p = 0.5$$

$$\begin{aligned}\mu_X &= 10 \cdot 0.5 \\ &= 5\end{aligned}$$

$$\begin{aligned}\sigma_X &= \sqrt{10 \cdot 0.5 \cdot 0.5} \\ &= 1.58\end{aligned}$$



New Distribution of Sample Proportions \hat{p}



$$\hat{p} = \text{---}$$

$$\mu_{\hat{p}} = \text{---}, \sigma_{\hat{p}} = \sqrt{\text{---}}$$