TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Population Standard Deviation (σ) Known

◆ To make a conf. int. for μ when σ is known, use $\bar{x}$ as a point estimate & margin of error:

New $E = z_{\alpha/2} \cdot \_\_\_\_$

► Recall: $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ has a standard normal distribution when X is normal ***OR*** sample size $n > 30$

EXAMPLE

Over 36 trips to work, you find a sample mean of 1 hour. Construct a 90% confidence interval for the true (population) mean travel time.

Use a population standard deviation of 18 minutes.

We are ___ % confident that the mean time to travel to work is between ______________ & ______________.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Find critical value: $z_{\alpha/2}$

3) Margin of Error: $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

4) Find upper & lower bounds

$(\bar{x} - E, \bar{x} + E)$

EXAMPLE

You consider how much time it takes to find a parking spot and find a mean of 1 hour and 22 minutes, with a population standard deviation of 15 minutes. Assume this information was based on a sample of 100 different days of circling the parking garage. Construct and interpret a 99% confidence interval.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE

Gas prices are getting more and more expensive. The mean gas price, from a random sample of 100 gas stations, was $3.50. It is assumed that gas prices have a population standard deviation of $0.04. Construct and interpret an 80% confidence interval for the true mean gas price in the United States.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Find critical value: $z_{\alpha/2}$

3) Margin of Error: $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

4) Find upper & lower bounds

$(\bar{x} - E, \bar{x} + E)$

PRACTICE

Books get more and more expensive every semester, but the distribution of their prices is always normal. 25 randomly selected students in your school spent a mean amount of $500 with a population standard deviation of $50. Construct and interpret a 98% confidence interval for the true mean spending on books.

PRACTICE

You want to purchase one of the new Altimas. You randomly select 400 dealerships across the United States and find a mean of $25,000. Assume a population standard deviation of $2500. Construct and interpret a 94% confidence interval for the true mean price for the new Nissan Altima.

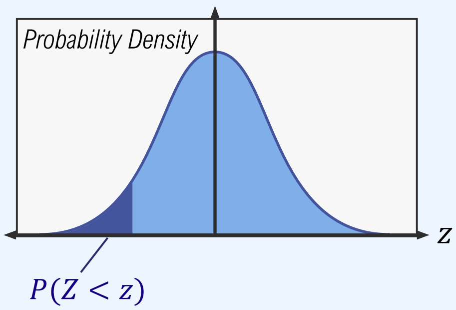
TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Critical Values: t-Distribution

◆ To create a conf. int. for μ when σ is unknown, use s to estimate σ. In this case, $\dfrac{\bar{X} - \mu}{/\sqrt{n}}$ follows a ***t*-distribution**.

► Finding a critical value $t_{\alpha/2}$ also requires **Degrees of Freedom**. Look up in a t-table or calculator.

Recall — **Normal Distribution**



$\mu_z = 0, \sigma_z = 1$

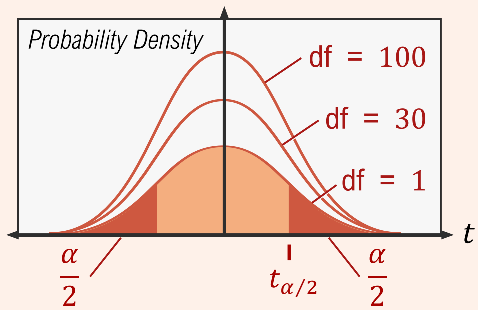
$\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

has a standard normal dist.

z-table: Use _______ to find $\boldsymbol{z}$

z	.00	...	.05	.06	.07	...
...	...	...	...	...	...	...
-2.0	.0228	...	.0202	.0197	.0192	...
-1.9	.0287	...	.0256	.0250	.0244	...
...	...	...	...	...	...	...

New — ***t*-Distribution**



$\mu_t = 0, \sigma_t \quad 1$

$\dfrac{\bar{X} - \mu}{s/\sqrt{n}}$

has t-distribution with _______ d.f.

t-table: Use _______ & _____ to find $\boldsymbol{t}$.

Conf. Lev.	50%	...	90%	95%	99%
↓ df | $\boldsymbol{\alpha/2}$ →	0.50	...	0.05	0.025	0.005
1	1.000	...	6.314	12.71	63.66
2	0.816	...	2.920	4.303	9.925
...	...	...	...	...	...
30	0.683	...	1.697	2.042	2.750

EXAMPLE

Find & compare the critical values for $(\boldsymbol{A})$ a 95% confidence interval with $\boldsymbol{\alpha}/2 = 0.025$ where σ is known & $(\boldsymbol{B})$ where σ is unknown, using a sample size of 31.

$(\boldsymbol{A})$

$(\boldsymbol{B})$

◆ You can only use $z_{\alpha/2}$ or $t_{\alpha/2}$ in a confidence interval if the sample is random & X is normal OR sample size $n > 30$.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE Answer the questions below.

(A) Find the critical value $t_{\alpha/2}$ for an 80% confidence interval given a sample size of 51.

(B) Find the critical value $t_{\alpha/2}$ for an 95% confidence interval given a sample size of 6.

EXAMPLE For each problem below determine if you can construct a confidence interval for μ using the standard normal or t-distribution. If so, which should be used? If not, why not?

(A) You randomly select 18 test scores from a Statistics class. The sample mean of the test scores is 74 and the population standard deviation is 6. Construct a 99% confidence interval for the population mean of the test scores.

(B) You randomly measure the height of 87 men and find an average height of 5'10" with a population standard deviation of 5 inches. Assuming the heights are normally distributed, construct a 99% confidence interval for the population mean height.

(C) 25 randomly selected students in your school spent on average $500 on textbooks with a sample standard deviation of $50. Assuming the spending on books is normally distributed, construct a 98% confidence interval for the population mean textbook spending.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE For which of the following scenarios can you NOT create a confidence interval using the standard normal or t-distribution?

$(\boldsymbol{A})$ In an attempt to see how their electric bill compares to their neighbors, a couple talks with 5 others on their street. They find that the sample mean monthly electric bill is $99.50 with a sample standard deviation of $9.20. Construct a 99% confidence interval for the population mean monthly electric bill.

$(\boldsymbol{B})$ Of 29 randomly selected rugby players, the sample mean resting heart rate is found to be 54 bpm with a population standard deviation of 3 bpm. Assuming the heart rates are normally distributed, construct an 80% confidence interval for the population mean resting heart rate.

$(\boldsymbol{C})$ 400 random gas stations are found to have a sample mean cost of $3.50 per gallon with a sample standard deviation of $0.50. Construct a 95% confidence interval for the true mean spending on books.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Using TI-84 for the t-Distribution

◆ To find a t-value ($t_{\alpha/2}$) from significance level (α), use the **4:invT** function.

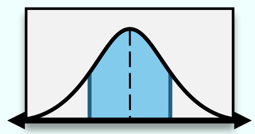
EXAMPLE Find the critical val. ($t_{\alpha/2}$) for an 80% conf. int. given $n = 51$.

$C =$ ________ $\alpha =$ _____ $\alpha/2 =$ _____

$df =$ __________

$-t_{\alpha/2} =$ ______ $t_{\alpha/2} =$ ______

Recall
$\alpha = 1 - C$
$df = n - 1$



HOW TO: Find t-Value From Prob. on TI-84

1) 2ND VARS (Distr), **4:invT**

2) Enter **area:** *(to left)*

df:

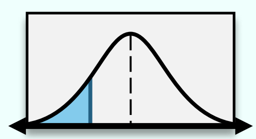
3) **paste**, ENTER

◆ To find a probability from t-value, use the **6:tcdf** function.

EXAMPLE Find $P(T < -1.47)$ given $df = 50$.

lower = __________

upper = __________



HOW TO: Find Prob. From t-Value on TI-84

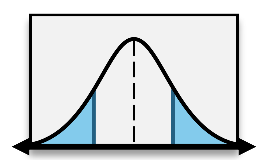
1) 2ND VARS (Distr), **6:tcdf**

2) Enter **lower:** (**-1E99** for -∞)

upper: (**1E99** for ∞)

df:

3) **paste**, ENTER



PRACTICE Fill out the table using a calculator and $n = 30$.

Conf Level (C)	Sig. Level (α)	Area to Left $\left(\frac{\alpha}{2}\right)$	$df = n - 1$	Critical Val. ($t_C = t_{\alpha/2}$)
90%	0.10			
95%	0.05			
99%	0.01			

HOW TO: Find t-Value From Prob. on TI-84

1) 2ND VARS (Distr), **4:invT**

2) Enter **area:** *(to left)*

df:

3) **paste**, ENTER

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE Find the critical val. ($t_{\alpha/2}$) for each confidence interval.

($\boldsymbol{A}$) Confidence Level $= 95\%, n = 35$

($\boldsymbol{B}$) Confidence Level $= 90\%, n = 48$

HOW TO: Find t-Value From Prob. on TI-84

1) 2ND VARS (Distr), `4:invT`

2) Enter `area:` *(to left)*

`df:`

3) `paste`, ENTER

PRACTICE Find each probability.

($\boldsymbol{A}$) $P(T > 1.5), df = 8$

($\boldsymbol{B}$) $P(-1.5 < T < 1.5), df = 8$

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Population Standard Deviation (σ) Unknown

◆ To create a confidence interval for μ when σ is unknown, use s & $t_{\alpha/2}$ instead of σ & $z_{\alpha/2}$ to find margin of error.

► Recall: $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ has a t-distribution with df = $n-1$, ONLY when X is normal ***OR*** sample size $n > 30$.

EXAMPLE

Over 36 trips to work, you find a sample mean of 1 hour & a sample standard deviation of 21 minutes. Construct & interpret a 90% confidence interval for the true (population) mean travel time.

We are ___ % confident that the mean time to travel to work is between _____________ & _____________.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Critical Val. (use table/calculator):

σ *known*: $z_{\alpha/2}$ ***OR*** *unknown*: ____

3) Margin of Error:

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ ***OR*** $E = t_{\alpha/2} \cdot \frac{\quad}{\sqrt{n}}$

4) Find upper & lower bounds

$(\bar{x} - E, \bar{x} + E)$

PRACTICE

You ask 16 people in your Statistics class what their grade is. The data appears to be distributed normally. You find a sample mean and sample standard deviation of 60 and 24, respectively. Construct and interpret a 95% confidence interval for the population mean class grade.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE

You want to take a trip to Paris. You randomly select 225 flights to Europe and find a mean and sample standard deviation of $1500 and $900, respectively. Construct and interpret a 95% confidence interval for the true mean price for a trip to Paris.

HOW TO: Make a Confidence Interval for μ (σ Known)

1) Verify: Sample is random ☐

X is normal ***OR*** $n > 30$ ☐

2) Critical Val. (use table/calculator):

σ *known*: $z_{\alpha/2}$ ***OR*** *unknown*: $t_{\alpha/2}$

3) Margin of Error:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \textbf{OR} \quad E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

4) Find upper & lower bounds

$$(\bar{x} - E, \bar{x} + E)$$

PRACTICE

You want to purchase one of the new Altimas. You randomly select 400 dealerships across the United States and find a mean of $25,000 and sample standard deviation of $2500. Construct and interpret a 94% confidence interval for the true mean price for the new Nissan Altima.

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

C.I. Using TI-84 – Mean

◆ To make a C.I. for the population mean using a calculator, use the `7:ZInterval` or `8:TInterval` function.

EXAMPLE Make a 95% C.I. for the mean in each data set.

(A)

$$\bar{x} = 23.67,\ \sigma = 5.724,\ n = 100$$

(B)

x-values							
21.7	18.42	25.3	13.85	19.1	22.68	16.7	24.59

HOW TO: Make a C.I. for Mean on TI-84

1) If given data, enter in **L1**

(STAT, **1:EDIT…**)

2) STAT > **TESTS**

3) If σ Known: **7: ZInterval**

If σ Unknown: **8: TInterval**

4)

If given data:	If given $\bar{x}, n, \ldots$:
DATA	**STATS**
σ: *(If Known)*	**σ:** *(If Known)* *or* **Sx:** *(If Unknown)*
List: L1	**$\bar{x}$:**
Freq: 1	**n:**
C-Level:	**C-Level:**

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

PRACTICE

A nutritionist wants to estimate the average grams of protein in a brand of protein bars. She takes a random sample of 40 protein bars with $\bar{x} = 210$ g & knows from prior data that $\sigma = 12$. Make a 95% conf. int. for μ.

We are ______% confident the protein bars have between (______ , ______) calories on average.

HOW TO: Make a C.I. for Mean on TI-84

1) If given data, enter in **L1** (STAT, **1:EDIT…**)

2) STAT > **TESTS**

3) If σ Known: **7: ZInterval**
If σ Unknown: **8: TInterval**

4)

If given data:	If given $\bar{x}, n, \ldots$:
DATA	**STATS**
σ: *(If Known)*	**σ:** *(If Known)* *or* **Sx:** *(If Unknown)*
List: L1	$\bar{x}$**:**
Freq: 1	**n:**
C-Level:	**C-Level:**

PRACTICE

To study the concentration of a particular pollutant (in parts per million) in a local river, an environmental scientist collects 32 water samples from random locations. They get $\bar{x} = 3.87$ ppm & know from previous data that $\sigma = 0.62$ ppm. Make a 99% conf. int. for the mean pollutant concentration.

We are ______% confident the mean pollutant concentration is between (______ , ______) ppm.

PRACTICE

A technician wants to estimate the average battery life of a new type of smart phone, so he tests 8 randomly selected phones & records the data below. Assuming battery life has a normal dist, make a 90% conf. int. for the mean battery life.

Battery Life (in Hours)							
12.5	13.1	12.8	13.4	12.9	13.0	13.2	12.7

TOPIC: CONFIDENCE INTERVALS FOR POPULATION MEAN

Topic Resource: Critical t-Values

Conf. Level	80%	90%	95%	98%	99%	99.9%
↓ df | $\alpha/2$ →	0.1	0.05	0.025	0.01	0.005	0.0005
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
...	...	...	...	...	...	...
40	1.303	1.684	2.021	2.423	2.704	3.551
50	1.299	1.676	2.009	2.403	2.678	3.496
60	1.296	1.671	2.000	2.390	2.660	3.460
70	1.294	1.667	1.994	2.381	2.648	3.435
80	1.292	1.664	1.990	2.374	2.639	3.416
90	1.291	1.662	1.987	2.368	2.632	3.402
100	1.290	1.660	1.984	2.364	2.626	3.390
...	...	...	...	...	...	...
∞	1.282	1.645	1.960	2.326	2.576	3.291