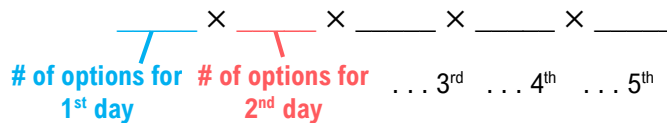


Introduction to Permutations

- For example, the different ways to wear 5 outfits for 5 days are permutations, because they are in **order**.



How many different way could you choose to get dressed over 5 days if you instead had 8 shirts?

_____ × _____ × _____ × _____ × _____

- ${}_nP_r$ or $P(n,r)$ can be read as: “number of permutations of **n** things taken **r** at a time”

New

$${}_nP_r = \frac{n!}{(n-r)!}$$

(Permutations)

(A) A teacher is choosing a line leader & a door holder from her class of 25 students. How many ways are there for the teacher to choose these positions?

(B) On your homework, there are 10 fill-in-the-blank questions with a word bank of 14 words. If you can only use a word once, how many possible ways could you answer these 10 questions?

◆ Solve permutation problems easily by rewriting the numerator to _____ the denominator.

TOPIC: COUNTING

PRACTICE

A student formed a club at their school. They have 13 members, and need to elect a president, vice president, and treasurer. How many ways are there to fill these officer positions?

PRACTICE

Emily is organizing her closet. She has 15 shirts left to hang but has space in one section for 6 shirts. How many ways could she hang shirts in that section?



PRACTICE

Evaluate the given expression. ${}_9P_4$

TOPIC: COUNTING

Permutations of Non-Distinct Objects

◆ When finding permutations, you may be faced with multiple objects that are the _____ or *non-distinct*.

Recall	Permutations of <i>Distinct</i> Objects	New	Permutations of <i>Non-Distinct</i> Objects
	<p>How many outcomes are there if you draw all marbles out of a bag of 6 different colored marbles?</p>  <div>${}_nP_r = \frac{n!}{(n-r)!}$$n = \text{total \# of objects}$$r = \text{\# of selected objects}$</div>		<p>How many outcomes are there if you draw all marbles out of a bag of 4 blue & 2 red marbles?</p>  <div>$\frac{n!}{r_1! r_2! r_3! \dots}$$n = \text{total \# of objects}$$r_1 = \text{\# of same objects of type 1}$$r_2 = \text{\# of same objects of type 2}$$r_3 = \text{\# of same objects of type 3}$</div>

◆ For permutations with non-distinct objects, rewrite numerator to cancel _____ factorial in the denominator.

EXAMPLE

(A) How many different 8-digit codes can be made using 5 zeros & 3 ones?

(B) How many ways are there to arrange the letters of BANANA?

TOPIC: COUNTING

PRACTICE

You want to arrange the books on your bookshelf by color. How many different ways could you arrange 12 books if 4 of them have a blue cover, 3 are yellow, and 5 are white?

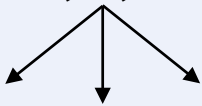
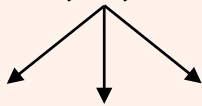
PRACTICE

How many ways are there to arrange the letters in the word CALCULUS?

TOPIC: COUNTING

Permutations vs. Combinations

- ◆ In math, permutations & combinations are different ways by which we organize objects.
 - Distinguish between the two by considering whether or not the _____ of objects matters.

Recall	Permutations	New	Combinations
	<p>Permutations of 2 letters from A, B, C</p>  <ul style="list-style-type: none">▸ A way to [GROUP ARRANGE] objects▸ Order [DOES DOES NOT] matter		<p>Combinations of 2 letters from A, B, C</p>  <ul style="list-style-type: none">▸ A way to [GROUP ARRANGE] objects▸ Order [DOES DOES NOT] matter

EXAMPLE Determine if the problem involves a **permutation** or **combination**. Do not solve.

<p>(A) An ice cream shop has 32 flavors. You need to pick 2 flavors to blend into a milkshake. How many possible ways can you select your flavors?</p> <p>[PERMUTATION COMBINATION]</p>	<p>(B) How many ways could a photographer line up the members of a family of 5?</p> <p>[PERMUTATION COMBINATION]</p>	<p>(C) How many different teams of 4 people can be formed from a group of 9 people?</p> <p>[PERMUTATION COMBINATION]</p>
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TOPIC: COUNTING

Combinations

- ◆ To calculate the number of **combinations** of r objects out of a group of n , divide the **permutations** by ____
 - ${}_nC_r$ or $C(n, r)$ can be read as: “number of combinations of n things taken r at a time”

Recall	Permutations	New	Combinations
	Permutations of 2 letters from A, B, C AB AC BC BA CA CB		Combinations of 2 letters from A, B, C AB AC BC
	${}_nP_r = \frac{n!}{(n-r)!} \longrightarrow {}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!}$		${}_nC_r = \frac{n!}{(n-r)! \cdot \underline{\hspace{1cm}}} \longrightarrow {}_3C_2 = \frac{3!}{(3-2)! \cdot \underline{\hspace{1cm}}} = \frac{3!}{\underline{\hspace{1cm}}}$

- ◆ Recall: When there are multiple factorials in denominator, rewrite numerator to cancel *highest* factorial in denom.

EXAMPLE

- (A) An ice cream shop has 32 flavors. You need to pick 2 flavors to blend into a milkshake. How many possible ways can you select your flavors?
- (B) How many different teams of 4 people can be formed from a group of 9 people?

TOPIC: COUNTING

PRACTICE

From a class of 28 students, in how many ways could a teacher select 4 students to lead the class discussion?

PRACTICE

Evaluate the given expression. ${}_{11}C_7$

TOPIC: COUNTING

EXAMPLE

When playing a particular lottery game, you must choose 5 numbers between 1 and 40. You win if the 5 numbers you choose match those drawn in the lottery. What is the probability that you will win if you purchase...

(A) 1 lottery ticket?

Recall

$$P(\text{event}) = \frac{\# \text{ of outcomes with event}}{\# \text{ of TOTAL outcomes}}$$

(Probability)

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

(Combinations)

(B) 50 different lottery tickets?