TOPIC: DISCRETE RANDOM VARIABLES

Intro to Random Variables & Probability Distributions

- A **Random Variable** represents a single number determined by __________ for each outcome of an "experiment".
  - ► Discrete (DRV): #'s **[CANNOT | CAN]** be broken down further [e.g. Dice roll]
  - ► Continuous (CRV): #'s **[CANNOT | CAN]** be broken down further [e.g. height]
- A **Probability Distribution** shows the probabilities of ______ possible values that a random variable can be.

EXAMPLE Verify that the table meets the criteria for a probability distribution.

Recall **Freq. Distribution**

Cans of soda drank per Day	Freq. f
0	10
1	20
2	40
3	20
4	10

New **Probability Distribution**

# Prizes won in random raffle X	Probability. $P(X)$
0	0.10
1	0.20
2	0.40
3	0.20
4	0.10

Criteria:

1) $\_\_\_ \leq P(X) \leq \_\_\_$ For any X

2) $\sum_{All\ X} P(X) = \_\_\_\_ = \_\_\_\_\%$

EXAMPLE You pay $1 to play a random lottery. The profits and probabilities of each outcome are as follows.

(A) What is the missing probability in the table?

(B) What is the probability of **at least** breaking even?

Lottery Profits	
Profit	Probability
-$1.00	0.40
$0.00	0.35
$5.00	?
$1,000,000.00	0.01

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PRACTICE A student is analyzing different types of variables in a statistics class. Which of the following below is a discrete random variable?

(A) The time it takes for a randomly selected runner to complete a 5K race
(B) The weight of a randomly chosen bag of apples from a grocery store
(C) The number of defective lightbulbs from a randomly chosen batch in a factory
(D) The number of days in a random month

EXAMPLE In a random survey, people were asked how many sodas they drink per day.

$(\boldsymbol{A})$ What is the probability a person responds with having **at most** 2 sodas per day?

Sodas per Day	Probability
0	0.50
1	0.31
2	0.09
3	0.05
4	0.03
5	0.01
6	0.01

$(\boldsymbol{B})$ What is the probability a person responds with having **between 1 and 4** sodas per day?

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Mean (Expected Value) of Random Variables

◆ To find the mean μ or "**Expected Value**" $E(X)$ of a DRV, __________ each value by its prob., then _____ results.

► If not given a table, make one and include a column for $X \cdot P(X)$.

EXAMPLE The table shows a probability distribution for the number of kids per household in a town. Find the expected value of this distribution.

# of kids (X)	**0**	**1**	**2**
Probability $P(X)$	0.15	0.60	0.25

Recall — **Mean of Data Set**

Sodas per day
0
1
2

$$\mu = \frac{\sum x}{N}$$

$$\frac{0+1+2}{3} = 1$$

New — **Mean of DRV with Probability Distribution**

X	$P(X)$	$X \cdot P(X)$
0	0.15	
1	0.60	
2	0.25	
Exp. Val. $E(X)$:		

$$\mu = E(X) = \sum$$

PRACTICE A factory produces lightbulbs in batches of 50. The probability distribution for the number of defective lightbulbs in a randomly selected batch is shown below. Find the expected value.

# of Defective bulbs (X)	**0**	**1**	**2**	**3**	**4**	**5**
Probability $P(X)$	0.20	0.30	0.25	0.15	0.07	0.03

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Variance & Standard Deviation of Discrete Random Variables

- To find variance (σ^2) and standard deviation (σ) of a DRV, make a table with columns for $X \cdot P(X)$ & $X^2 \cdot P(X)$:
  - Recall: To find standard deviation, $\sigma = \sqrt{\sigma^2}$

New

$$\sigma^2 = \sum[X^2 \cdot P(X)] - \mu^2$$

$$= \sum[(X-\mu)^2 \cdot P(X)]$$

← Easier to use!

EXAMPLE

The table shows a probability distribution for the number of kids per household in a town. Find the variance and standard deviation of this distribution.

# of kids (X)	0	1	2
Probability $P(X)$	0.15	0.60	0.25

New — **Variance & Standard Deviation of DRV**

X	$P(X)$	$X \cdot P(X)$	$X^2 \cdot P(X)$
0	0.15	$0 \cdot 0.15 = 0$	
1	0.60	$1 \cdot 0.60 = 0.60$	
2	0.25	$2 \cdot 0.25 = 0.50$	
		$\mu = 1.10$	

Mean (μ): ____________

μ^2: ____________

$\sum X^2 \cdot P(X)$: ____________

Variance (σ^2): ____________

Std. Dev. (σ) : ____________

PRACTICE

A company tracks the number of complaints they receive, where the random variable X is the number of complaints received daily. Find the variance & standard deviation of this distribution.

# of complaints (X)	0	1	2	3
Probability $P(X)$	0.45	0.30	0.20	0.05

Recall

$$\sigma^2 = \sum[X^2 \cdot P(X)] - \mu^2$$